

Lecture 7  
2022/2023

# Microwave Devices and Circuits for Radiocommunications

# 2022/2023

- 2C/1L, **MDCR**
- Attendance at minimum 7 sessions (course or laboratory)
- Lectures- **associate professor Radu Damian**
  - Tuesday 12-14, ~~Online~~, P8
  - E – 50% final grade
  - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 21-28.02.2023 (t<sub>2</sub> and t<sub>3</sub> not announced, lecture)
    - 3att.=+0.5p
  - all materials/equipments authorized

# 2022/2023

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 21.02.2022)
    - personal homework

# Materials

■ <http://rf-opto.etti.tuiasi.ro>

The screenshot shows a web browser displaying the website [http://rf-opto.etti.tuiasi.ro/microwave\\_cd.php?ch\\_lang=0](http://rf-opto.etti.tuiasi.ro/microwave_cd.php?ch_lang=0). The page title is "Microwave Devices and Circuits for Radiocommunications (English)". The main content area includes sections for Course (MDCR 2017-2018), Activities, Evaluation, Grades, Attendance, Lists, and Materials. The right side features the RF-OPTO logo, a globe graphic, and language links (English, Romana). A red circle highlights the "English" link in the language bar.

Laboratorul de Microunde și Optică

Main Courses Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

## Microwave Devices and Circuits for Radiocommunications (English)

**Course:** MDCR (2017-2018)

**Course Coordinator:** Assoc.P. Dr. Radu-Florin Damian  
**Code:** EDOS412T  
**Discipline Type:** DOS; Alternative, Specialty  
**Credits:** 4  
**Enrollment Year:** 4, Sem. 7

**Activities**

**Evaluation**

Type: Examen

A: 50%, (Test/Colloquium)  
B: 25%, (Seminary/Laboratory/Project Activity)  
D: 25%, (Homework/Specialty papers)

**Grades**

[Aggregate Results](#)

**Attendance**

[Course](#)  
[Laboratory](#)

**Lists**

[Bonus-uri acumulate \(final\)](#)  
[Studentii care nu pot intra in examen](#)

**Materials**

**Course Slides**

[MDCR Lecture\\_1 \(pdf, 5.43 MB, en, !\[\]\(5bd3139e49b8ec618dddaa46174de8b0\_img.jpg\)](#)  
[MDCR Lecture\\_2 \(pdf, 3.67 MB, en, !\[\]\(9aae4ef11f04080694e1bcd3250dc654\_img.jpg\)](#)  
[MDCR Lecture\\_3 \(pdf, 4.76 MB, en, !\[\]\(1f875e8ff0db454eb302861a56ff194f\_img.jpg\)](#)  
[MDCR Lecture\\_4 \(pdf, 5.58 MB, en, !\[\]\(05604d380e755a92e3161ab249a7c58e\_img.jpg\)](#)

 ETI

**RF-OPTO**



 English |  Romana |

Main Courses Master Staff Research

Grades Student List Exams Photos

## Online Exams

In order to participate at online exams you must get ready following

# Materials

- RF-OPTO
  - <http://rf-opto.eti.tuiasi.ro>
- **David Pozar, “Microwave Engineering”,**  
Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by **email**/online exam
  - used at lectures/laboratory

# Access

- Not customized

A screenshot of a student profile page. On the left is a thumbnail photo of a student. Below it is a link "Acceseaza ca acest student". To the right is a section titled "Date:" containing the following information:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Below this is a section titled "Note obtinute" with a table:

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

A screenshot of a contact form. It includes fields for "Nume" (Name) with a redacted value, "Email" (Email), and "Cod de verificare" (Verification code) with a redacted value. At the bottom is a large blue button containing the text "344bd9f". A red arrow points from the "Email" field on the left to the verification code button on the right.

Nume  
MOOROUN

Email

Cod de verificare

344bd9f

Trimite

# Online

- access to **online exams** requires the **password** received by email

English | Romana |

Main Courses Master Staff Research **Student List**

Grades Student List Exams Photos

## POPESCU GOPO ION

Fotografia nu există

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telecomunicatii
Marca	7000000

[Access the site as this student](#) | [Request access to software](#)

**Grades**

Inca nu a fost notat.

Main Courses Master Staff Research

Grades **Student List** Exams Photos

### Login

Use the last name and email stored in the database

Name  
POPESCU GOPO

Email/Password

Write the code below

828f26b

Send

# Online

- access email/password

Main Courses Master Staff Research

Grades Student List Exams Photos

## POPESCU GOPO ION

**Fotografia nu există**

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telec
Marca	7000000

You access the site as **this student!**

Main Courses Master Staff Research

Grades Student List Exams Photos

## POPESCU GOPO ION

**Fotografia nu există**

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telec
Marca	7000000

You access the site as **this student (including exams)!**

# Password

## ■ received by email

Important message from RF-OPTO Inbox x

Radu-Florin Damian  
to me, POPESCU ▾

Romanian ▾ English ▾ Translate message

 Laboratorul de Microunde si Optoelectronica  
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei  
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul rf-opto este  
Parola: [REDACTED]

Identificati-vă pe [server](#), cu parola, cat mai rapid, pentru confirmare.

**Memorati** acest mesaj intr-un loc sigur, pentru utilizare ulterioara

---

Attention: POPESCU GOPO ION

The password to access the exams on the rf-opto server is  
Password: [REDACTED]

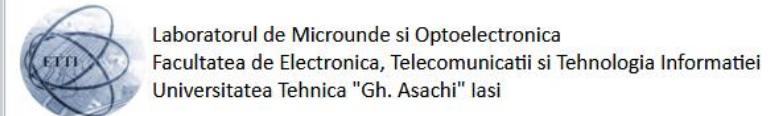
Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply Reply all Forward

Subject: Important message from RF-OPTO Correspondents: POPESCU GOPO ION

From: Me <[rdamian@etti.tuiasi.ro](mailto:rdamian@etti.tuiasi.ro)> ★  
Subject: Important message from RF-OPTO (highlighted)  
To: [REDACTED]  
Cc: Me <[rdamian@etti.tuiasi.ro](mailto:rdamian@etti.tuiasi.ro)> ★



In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul rf-opto este  
Parola: [REDACTED]

Identificati-vă pe [server](#), cu parola, cat mai rapid, pentru confirmare.

**Memorati** acest mesaj intr-un loc sigur, pentru utilizare ulterioara

---

Attention: POPESCU GOPO ION

The password to access the exams on the rf-opto server is  
Password: [REDACTED]

Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

# Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~

## Materials

### Other data

[Manual examen on-line \(pdf, 2.65 MB, ro, !\[\]\(8b57f0e15e7dda24cf9977561475f640\_img.jpg\)](#)

[Simulare Examen \(video\) \(mp4, 65.12 MB, ro, !\[\]\(4cafc60cd39da821525d7c6589540296\_img.jpg\)](#)

# Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~

Announcement 23:59 (10/05/2020)	Support material 00:05 (11/05/2020)	Exam Topics 00:07 (11/05/2020)	Results 00:10 (11/05/2020)	End 00:20 (15/05/2020)	Confirmation 00:20 (16/05/2020)	Next timeframe in: <b>05 m 43 s</b> <a href="#">Refresh now</a>
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**Announcement**

This is a "fake" exam, introduced to familiarize you with the server interface and to perform the necessary actions during an exam: thesis scan, selfie, use email for co...

**Server Time**

All exams are based on the server's time zone (it may be different from local time). For reference time on the server is now:

**10/05/2020 23:59:16**

# Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgromot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri S	T2, fisier parmetri S	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

# Online results submission

- many numerical values

i	z1	z2	z3	z4	z5	z6	z7
	148.33	155.88	202.12	164.35	180.91	30.29	185.19
	25.97	153.5	34.64	35.79	55.56	26.212	10,693
	0	0	0	0	0	0	0
	50	50	50	50	50	50	50



# Online results submission

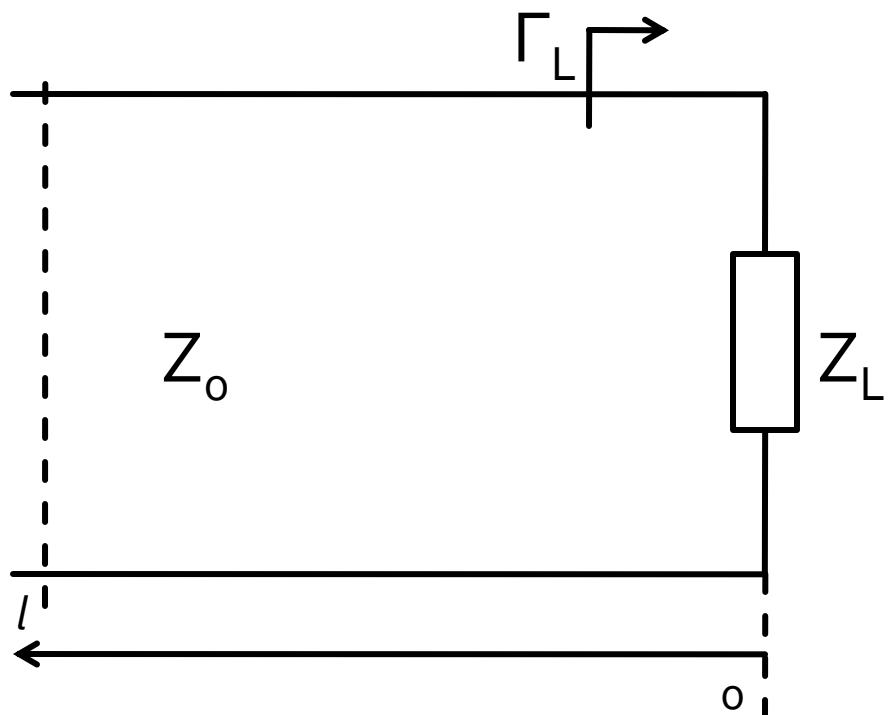
Grade = Quality of the work +  
+ Quality of the submission

# TEM transmission lines

# Course Topics

- **Transmission lines**
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

# The lossless line



$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- $Z_0$  real

# The lossless line

$$V(z) = V_0^+ \cdot (e^{-j\beta z} + \Gamma \cdot e^{j\beta z})$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta z} - \Gamma \cdot e^{j\beta z})$$

- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\left\{1 - \Gamma^* \cdot e^{-2j\beta z} + \Gamma \cdot e^{2j\beta z} - |\Gamma|^2\right\}$$

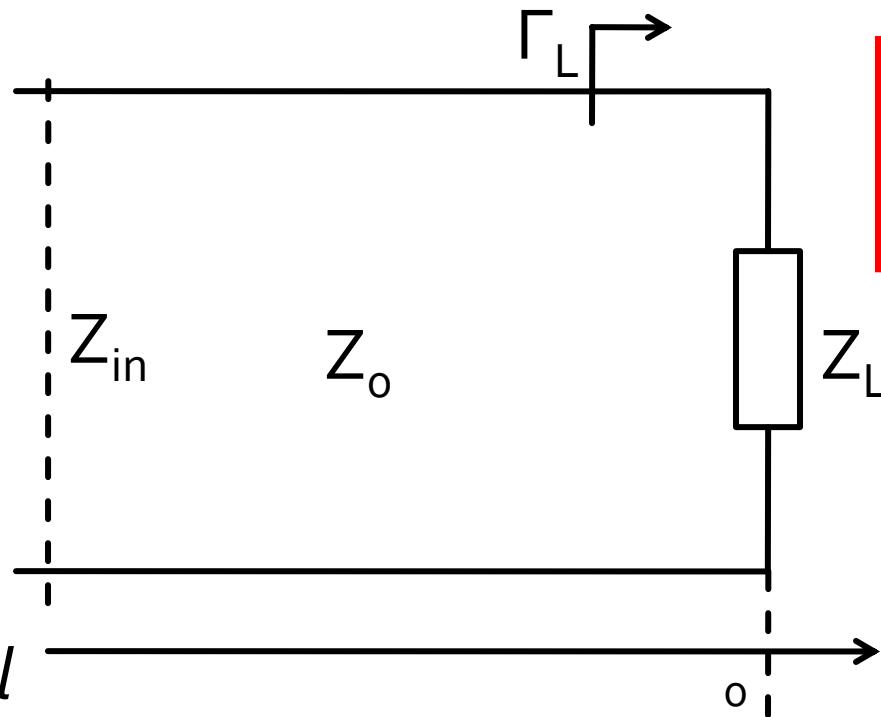
$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

$$(z - z^*) = \text{Im}$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB] 
$$\text{RL} = -20 \cdot \log|\Gamma| \quad [\text{dB}]$$

# The lossless line

- input impedance of a length  $l$  of transmission line with characteristic impedance  $Z_0$ , loaded with an arbitrary impedance  $Z_L$



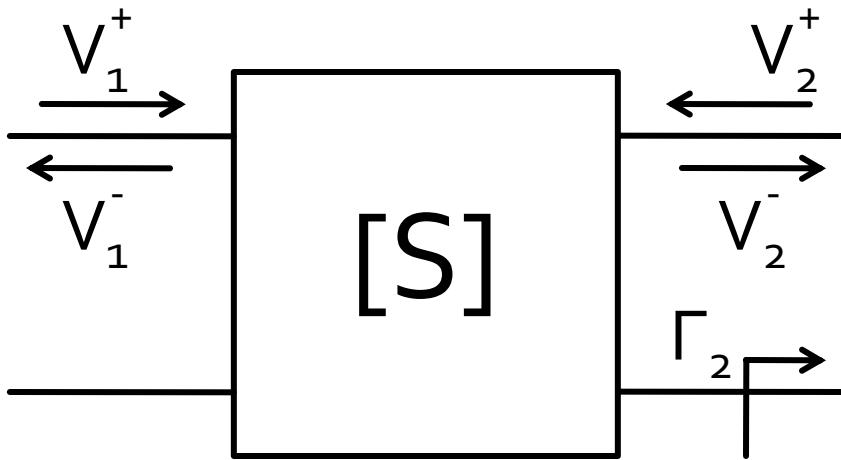
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

General theory

# Microwave Network Analysis

# Scattering matrix – $S$

## ■ Scattering parameters



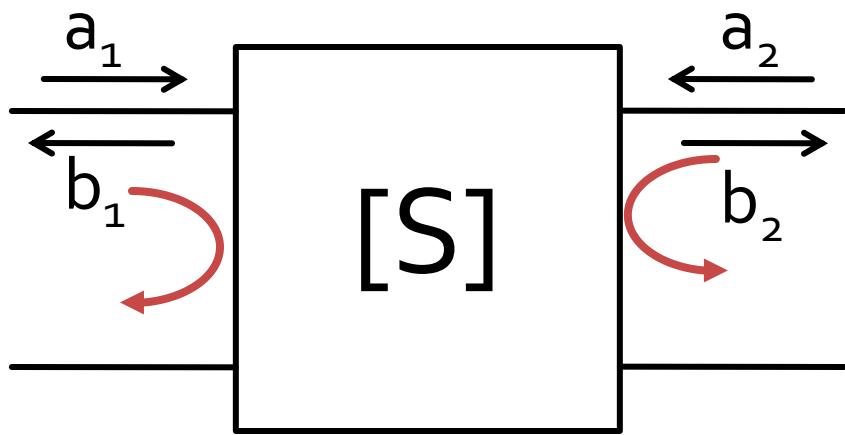
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} \quad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0}$$

- $V_2^+ = 0$  meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

# Scattering matrix – $S$

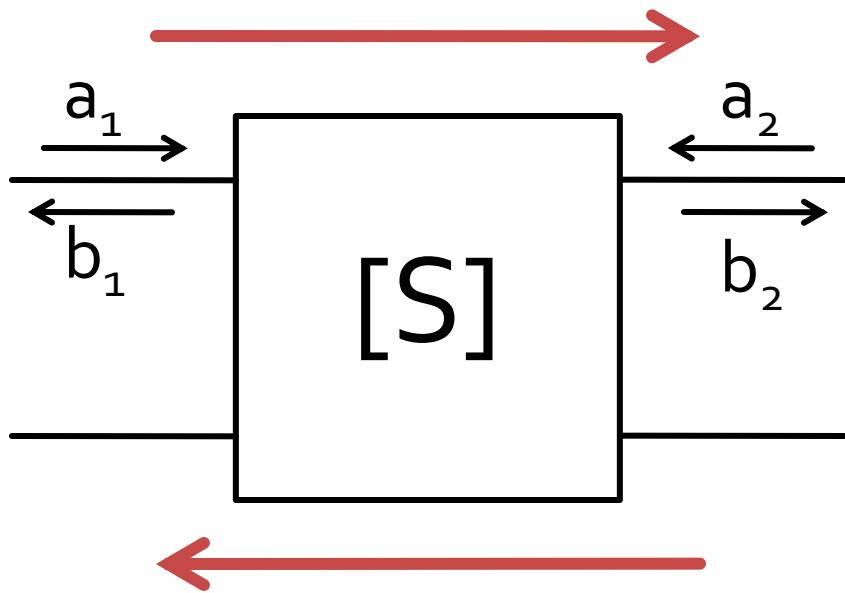


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

- $S_{11}$  and  $S_{22}$  are reflection coefficients at ports 1 and 2 when the other port is matched

# Scattering matrix – $S$



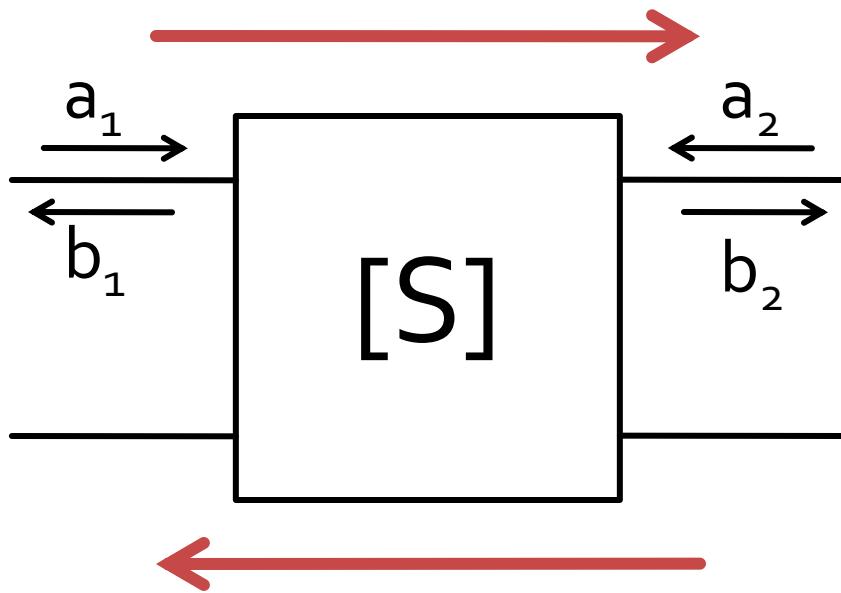
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \frac{b_2}{a_1} \Big|_{a_2=0}$$

$$S_{12} = \frac{b_1}{a_2} \Big|_{a_1=0}$$

- $S_{21}$  si  $S_{12}$  are signal amplitude gain when the other port is matched

# Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{\text{Power in } Z_0 \text{ load}}{\text{Power from } Z_0 \text{ source}}$$

- a,b
  - information about signal power **AND** signal phase
- $S_{ij}$ 
  - network effect (gain) over signal power **including** phase information

Impedance Matching

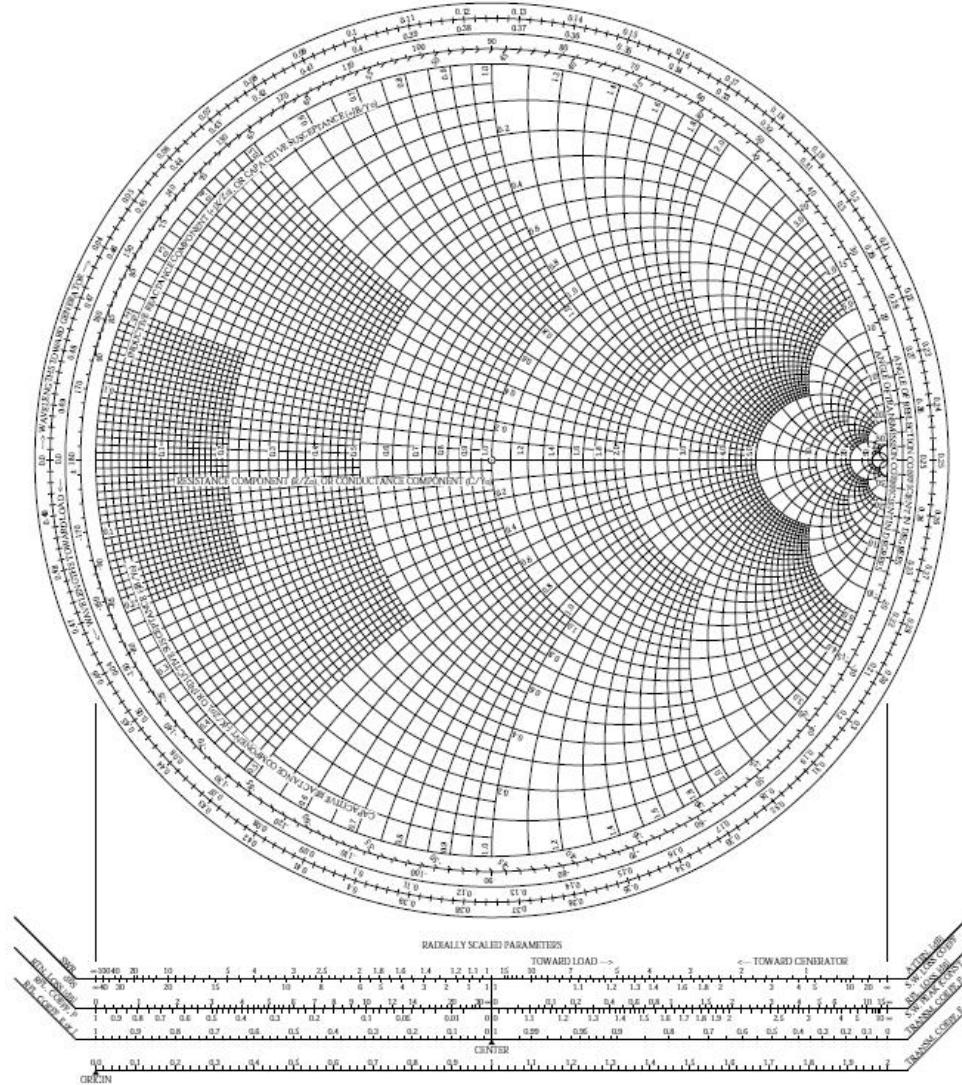
# The Smith Chart

# Course Topics

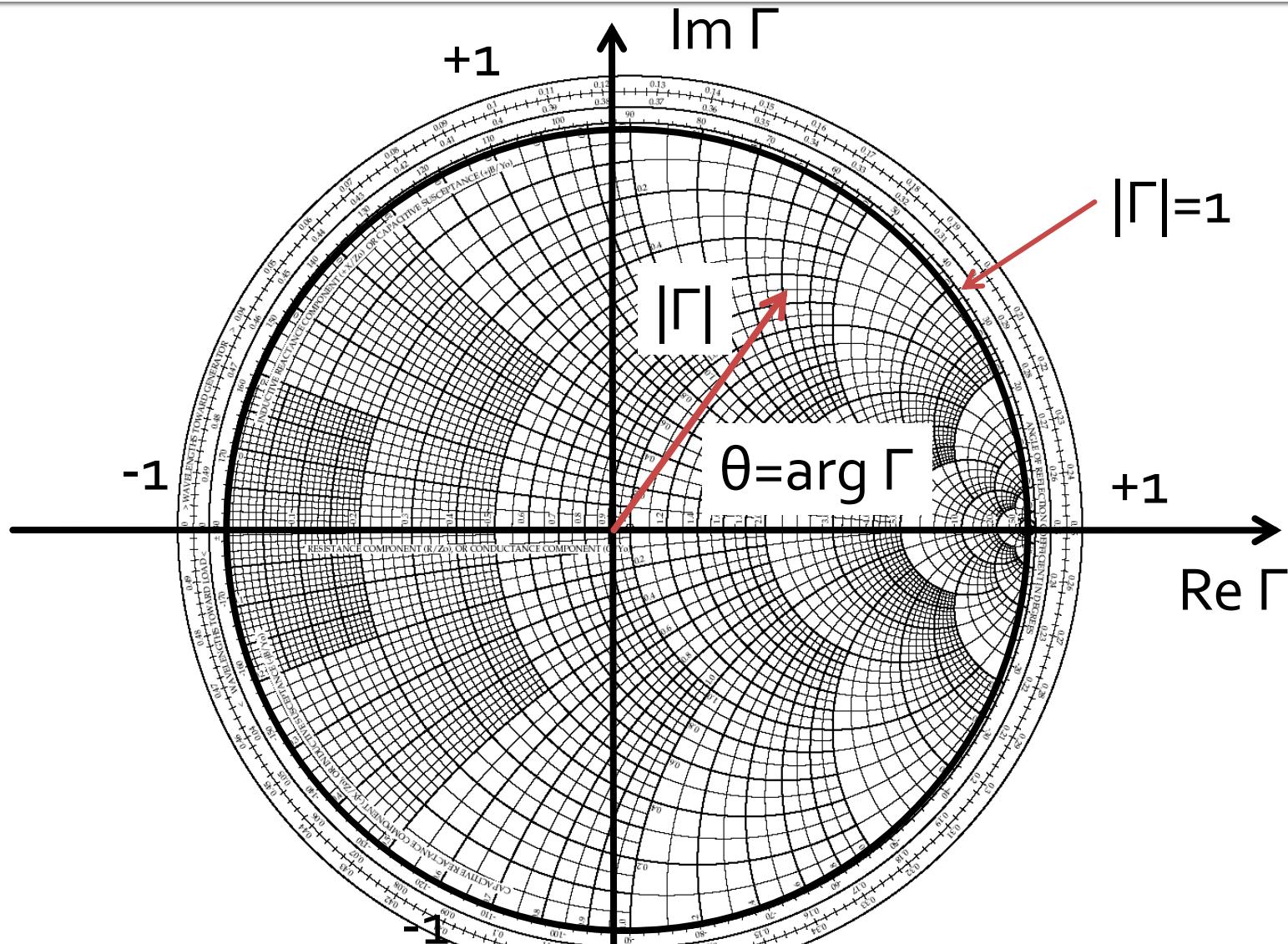
- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~



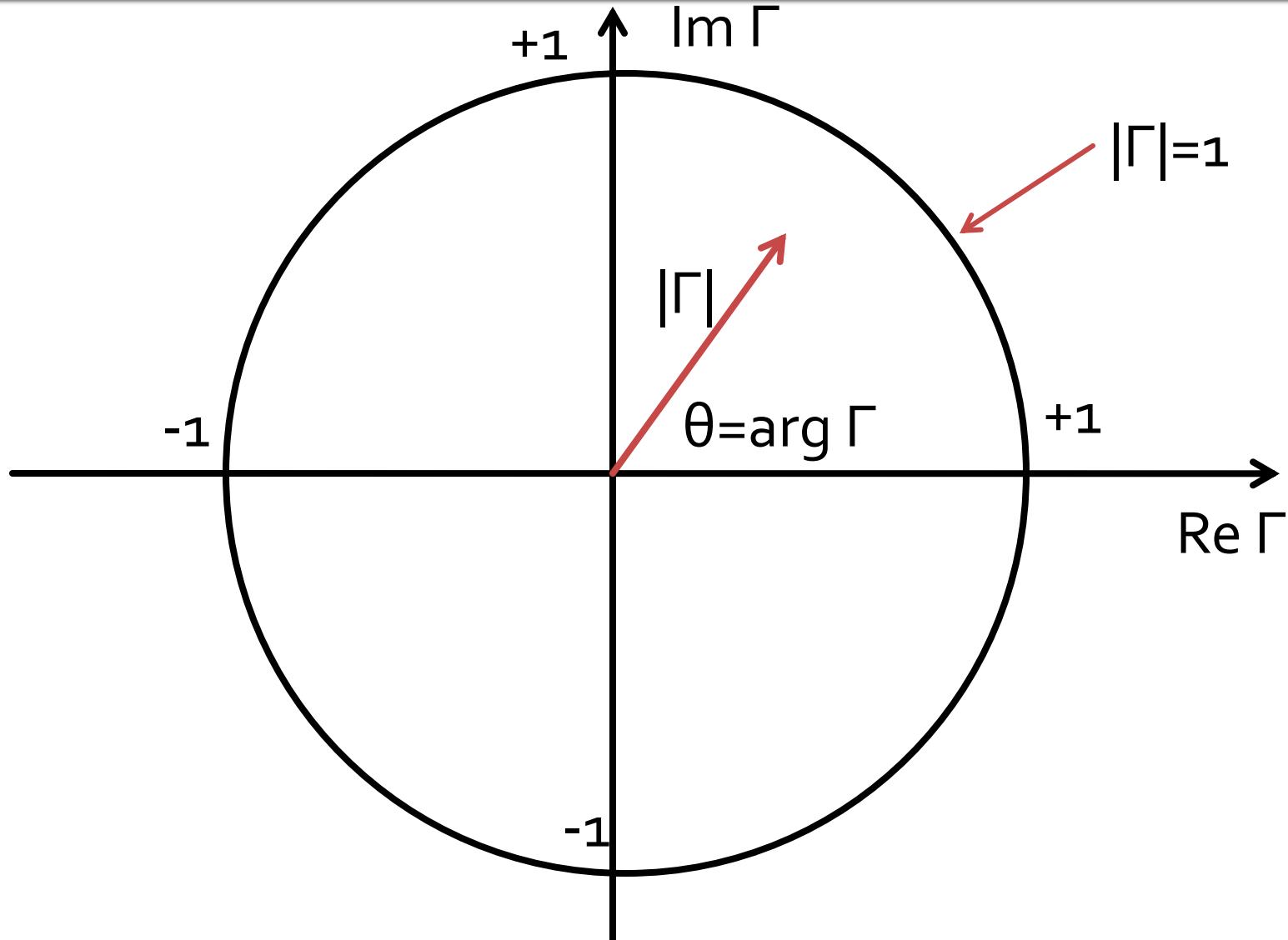
# The Smith Chart



# The Smith Chart

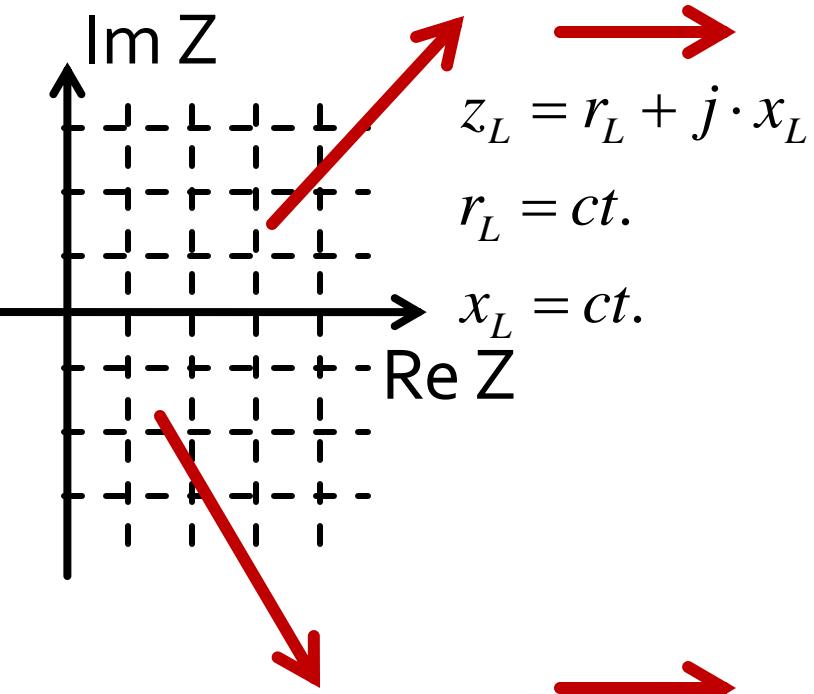


# The Smith Chart

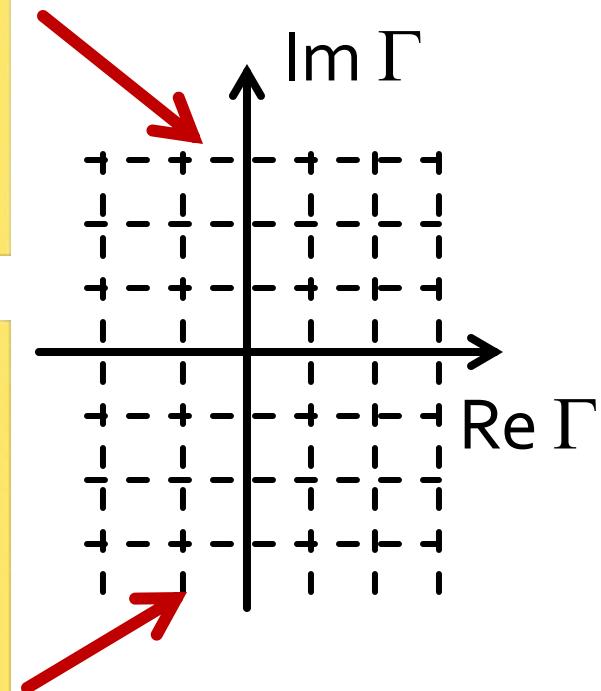
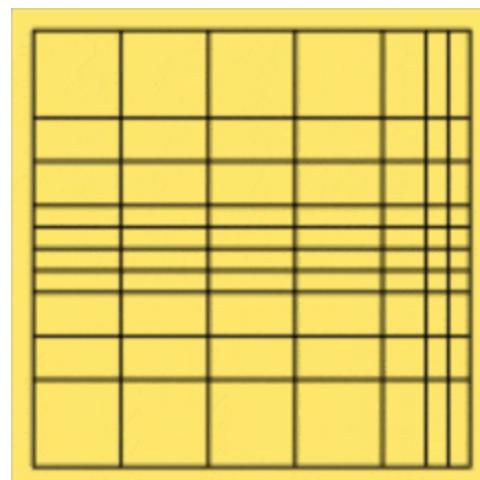
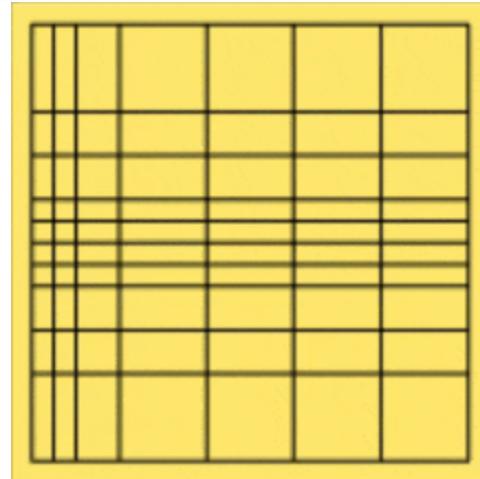


# The Smith Chart

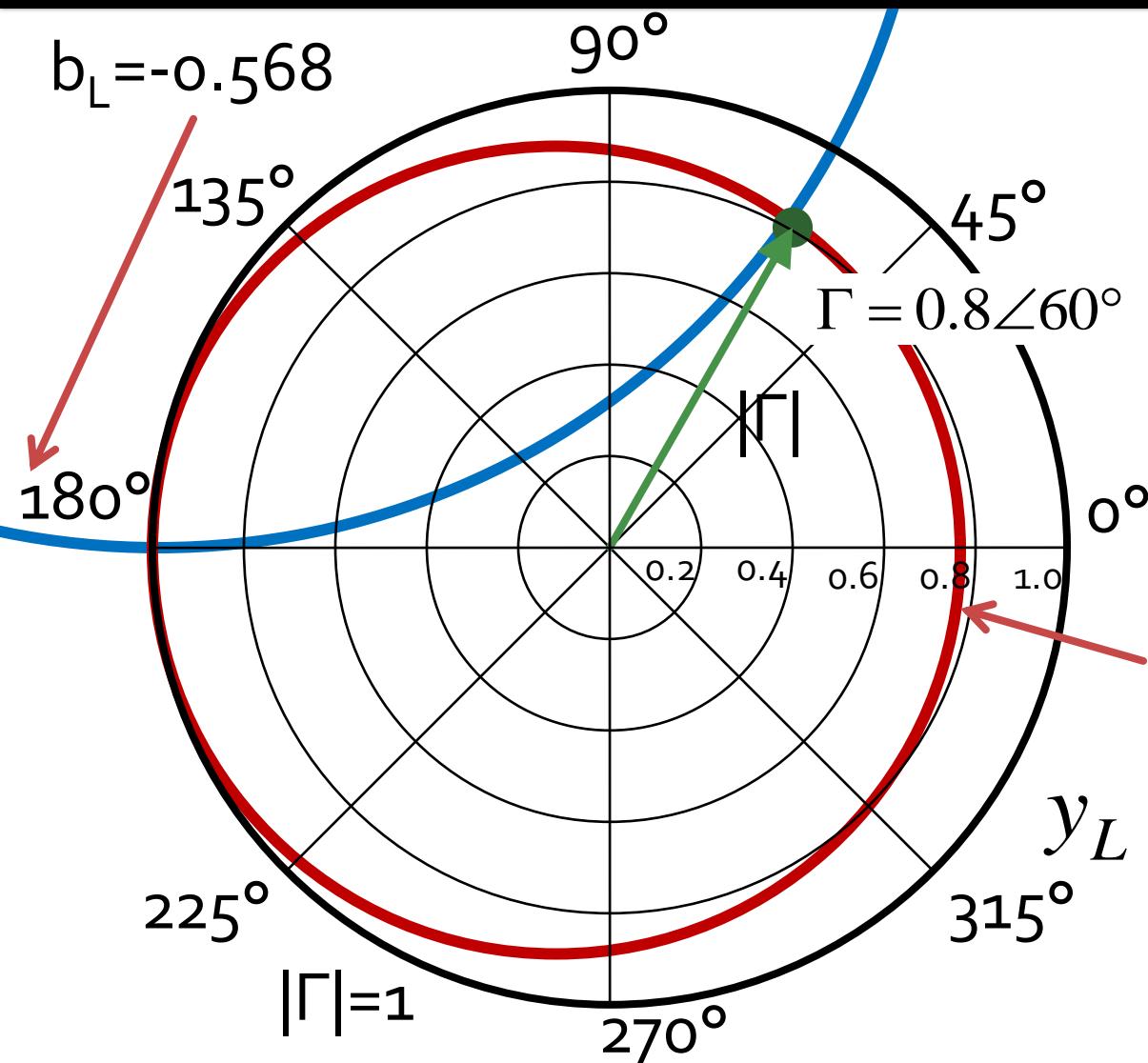
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$



# The Smith Chart, reflection coefficient $\Leftrightarrow$ admittance



$$\Gamma = 0.8 \angle 60^\circ$$

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

$$y_L = \frac{1}{z_L} = 0.148 - j \cdot 0.568$$

$$g_L = 0.148$$

$$y_L = 0.148 + j \cdot 0.568$$

(whatever  $Z_0$ )

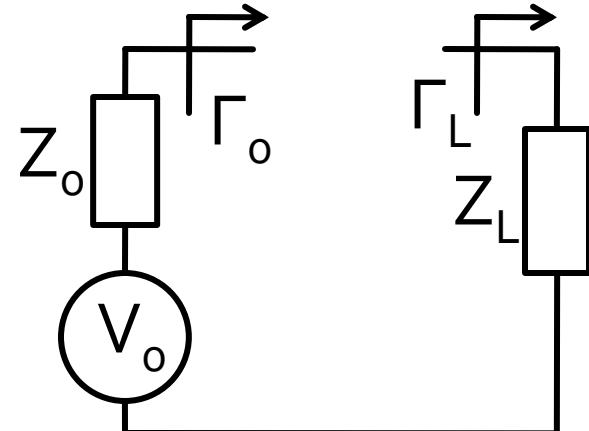
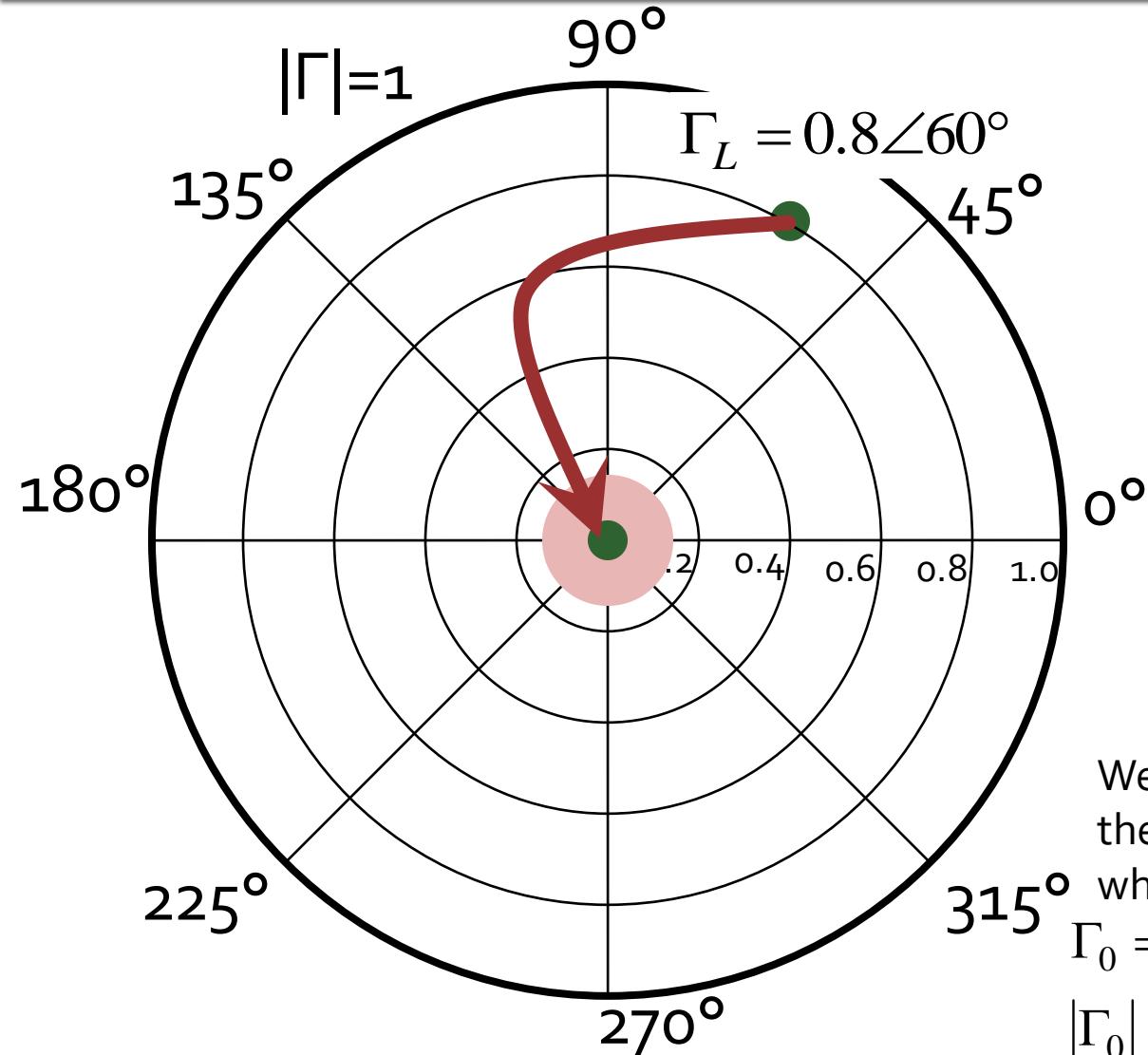
Impedance matching

# **Impedance Matching with lumped elements (L Networks)**

# Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

# The Smith Chart, reflection coefficient, impedance matching



Matching  $Z_L$  load to  $Z_0$  source.  
We normalize  $Z_L$  over  $Z_0$

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

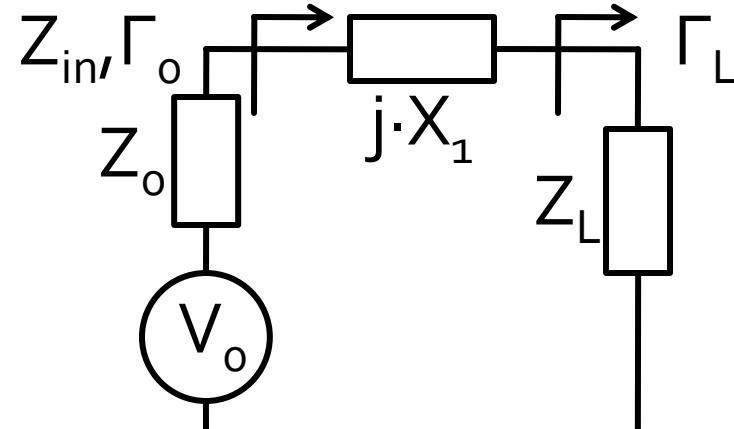
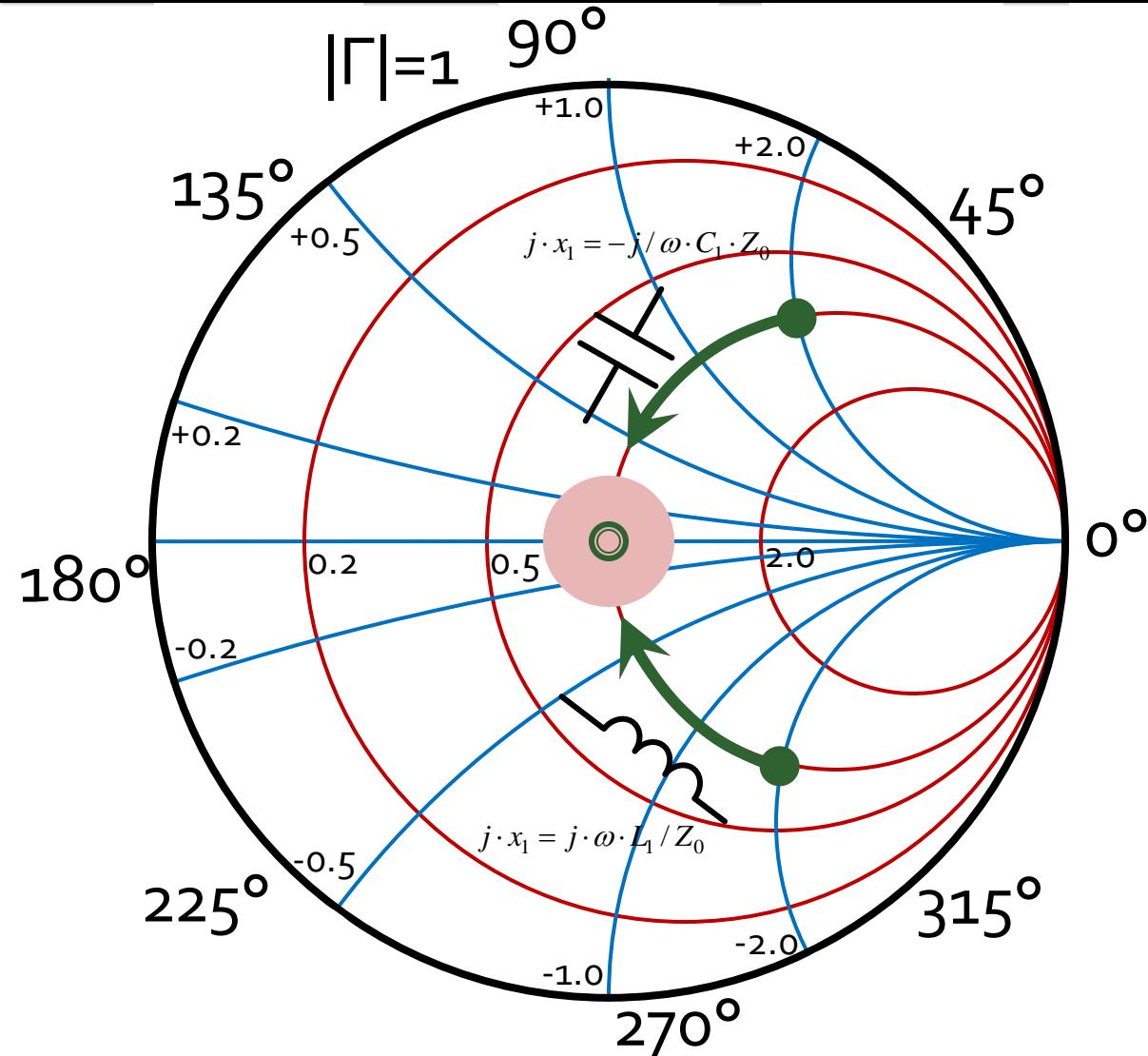
$$\Gamma_L = 0.8\angle 60^\circ$$

We must move the point denoting  
the reflection coefficient in the area  
where with a  $Z_0$  source we have:

$$\Gamma_0 = 0 \text{ perfect match } \bullet$$

$|\Gamma_0| \leq \Gamma_m$  "good enough" match

# Matching, series reactance



$$z_L = r_L + j \cdot x_L$$

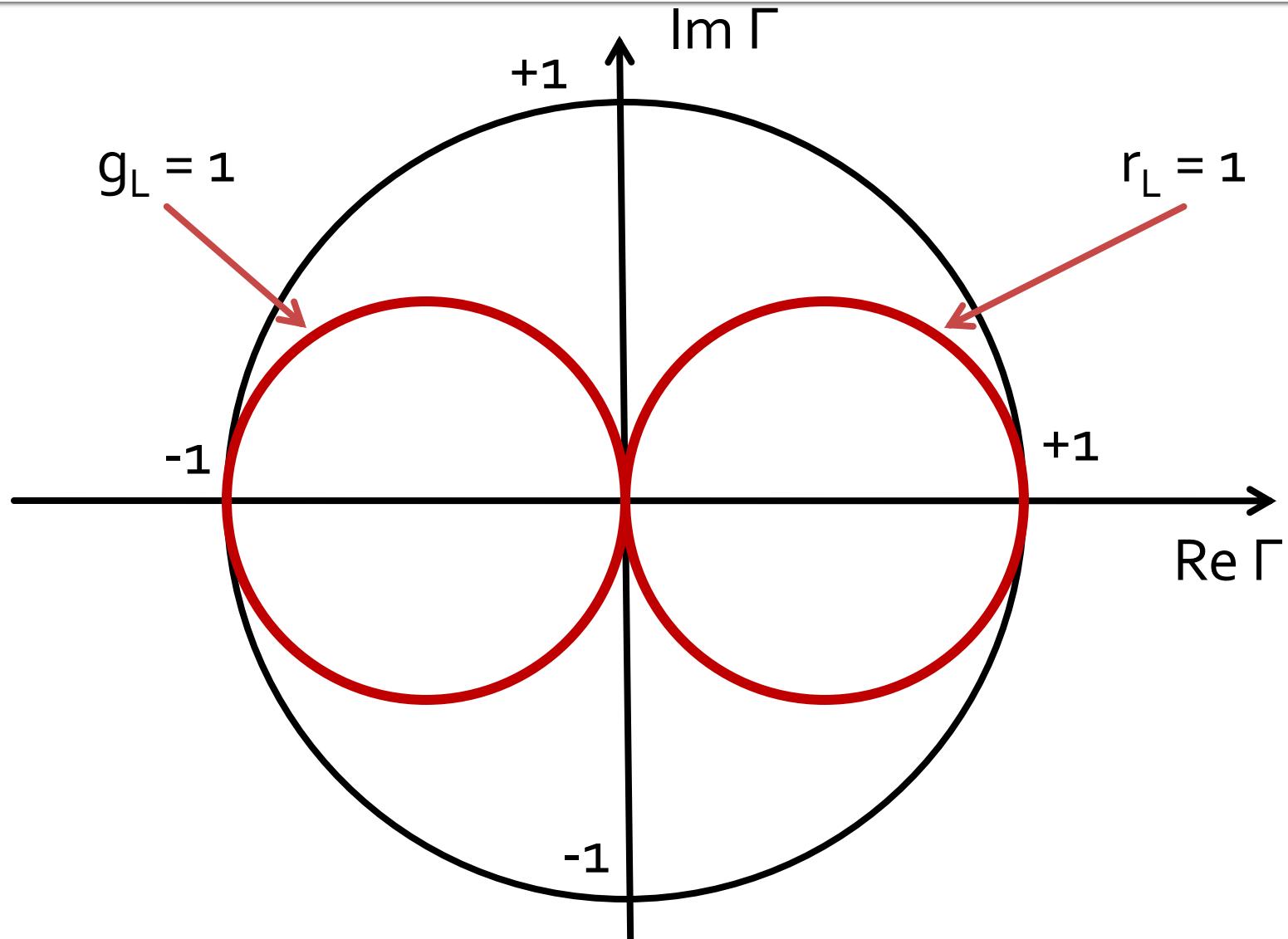
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

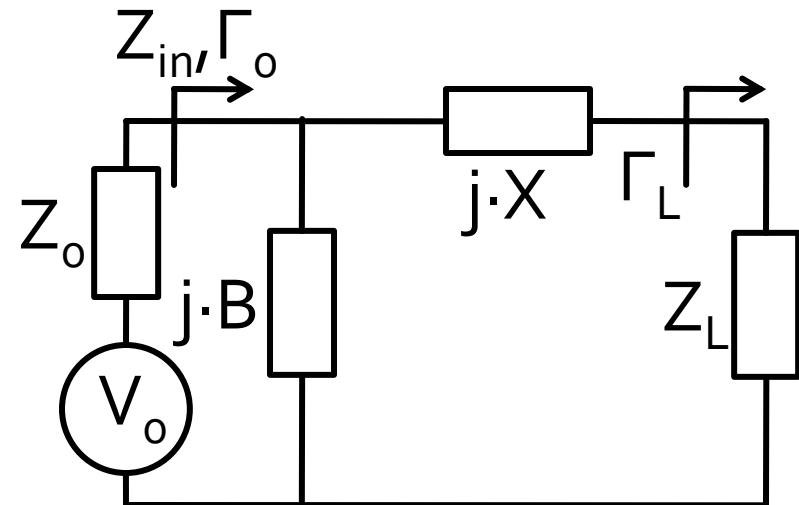
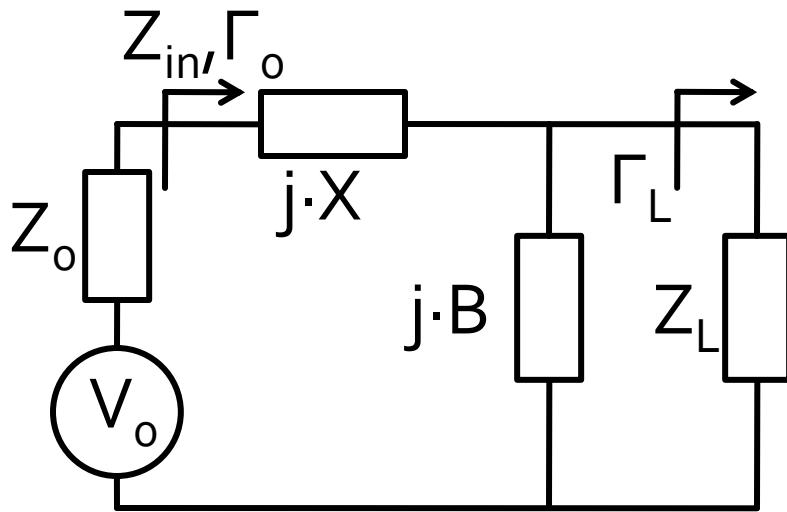
- Match can be obtained **if and only if**  $r_L = 1$
- we compensate the reactive part of the load

$$j \cdot x_1 = -j \cdot x_L$$

# Smith chart, $r=1$ and $g=1$

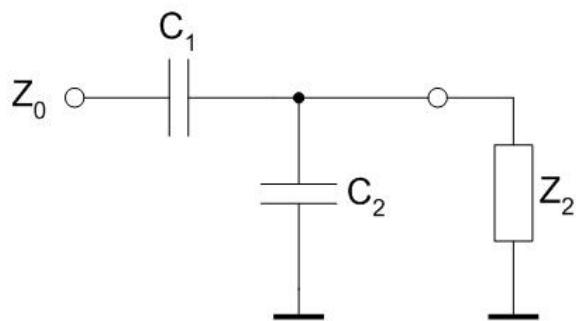
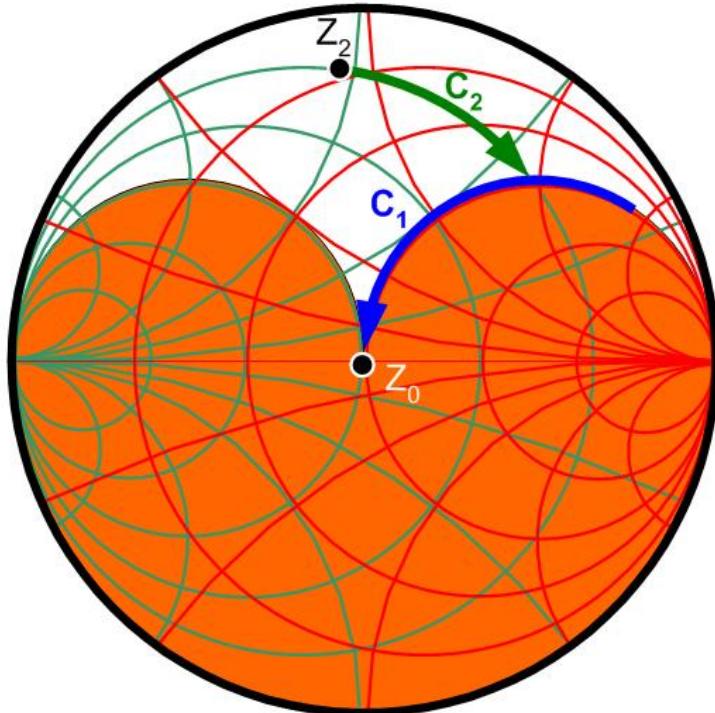


# Matching with 2 reactive elements (L Networks)

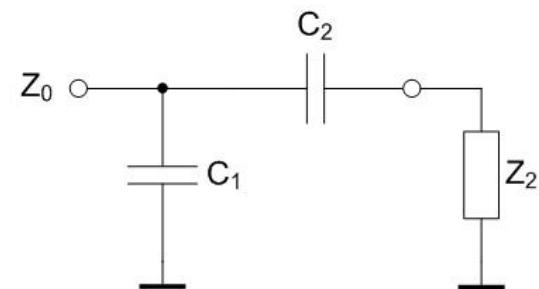
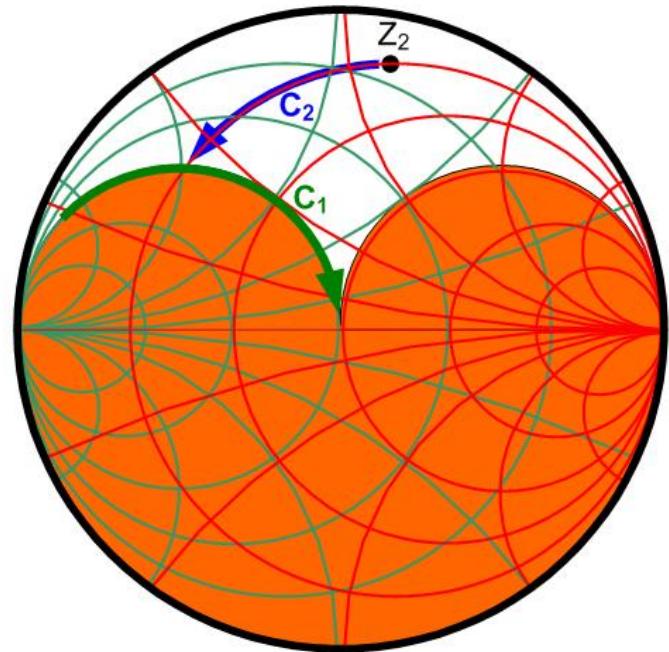


- Two steps matching
  - first reactive element moves the reflection coefficient **on the circle**  $r_L = 1/g_L = 1$
  - second element compensates the remaining reactance and achieves the impedance match

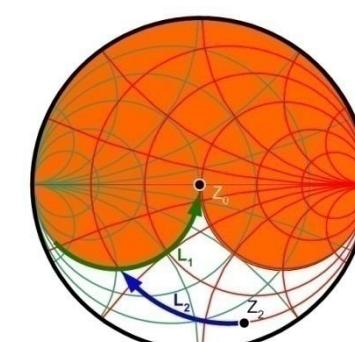
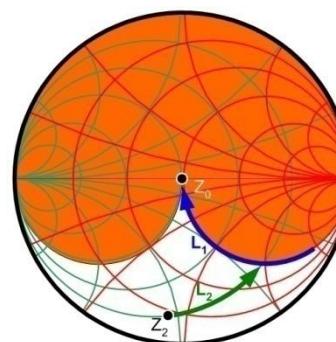
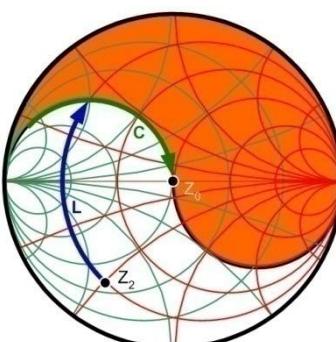
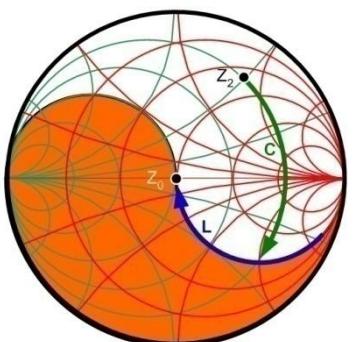
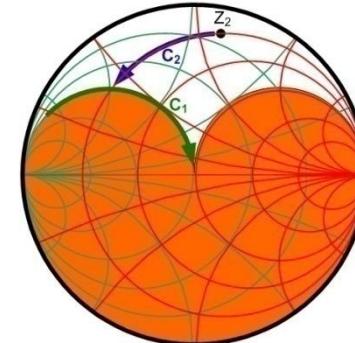
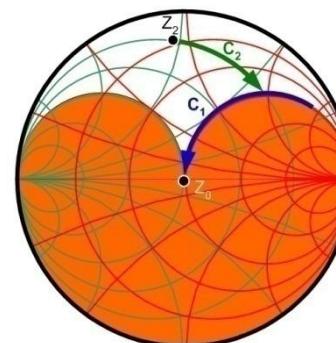
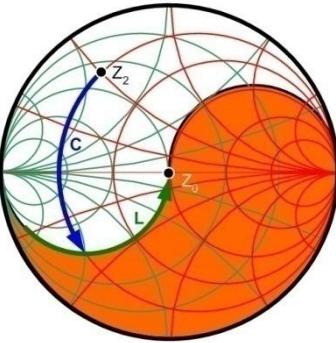
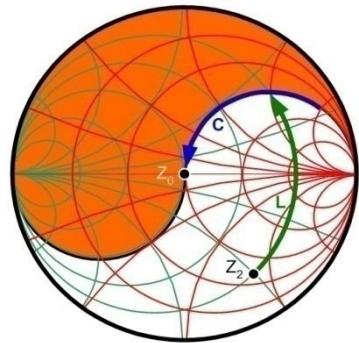
# series C, shunt C / shunt C, series C



Forbidden area for  
current network



# Matching with 2 reactive elements (L Networks)

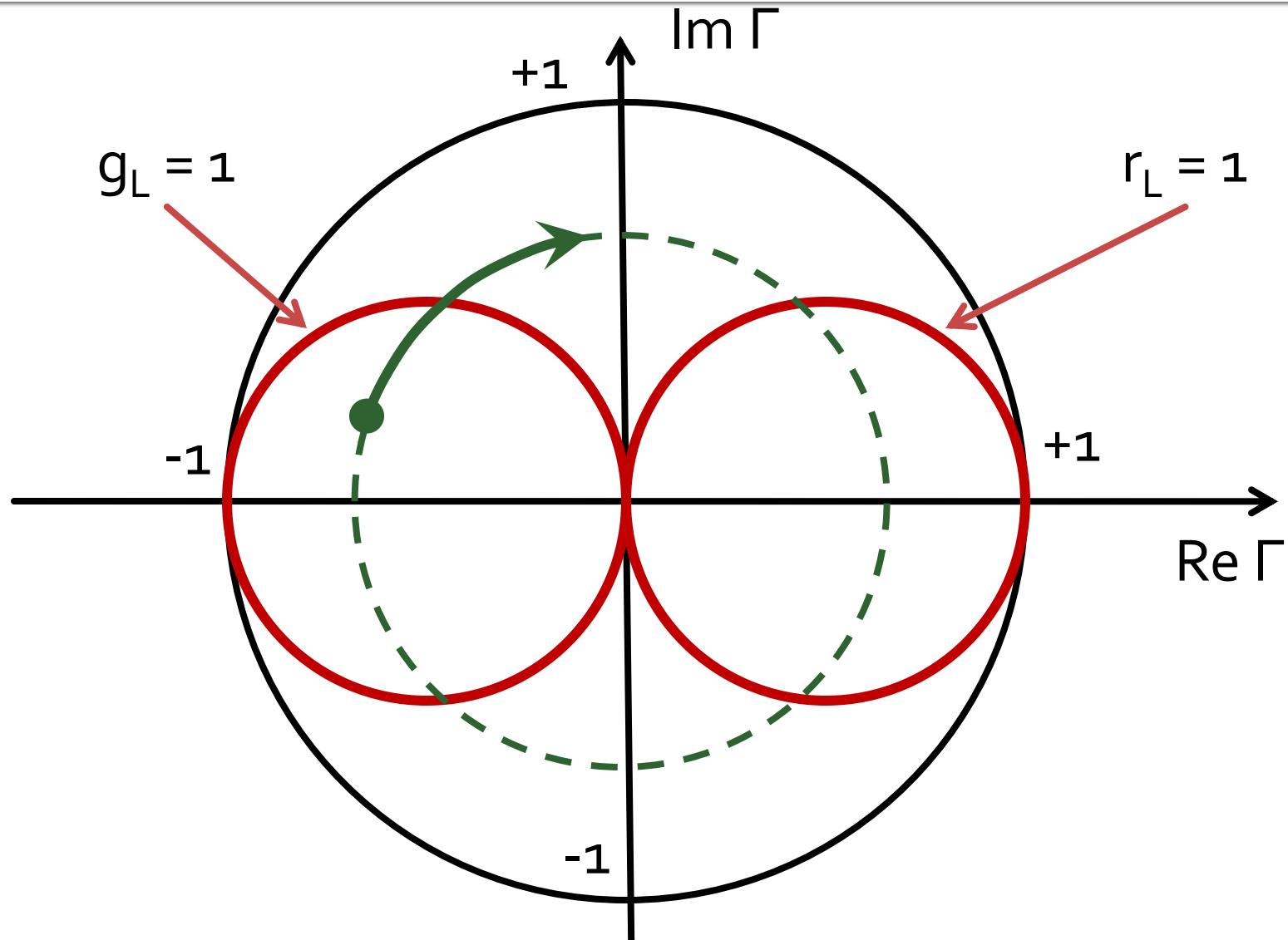


Forbidden area for  
current network

Impedance Matching

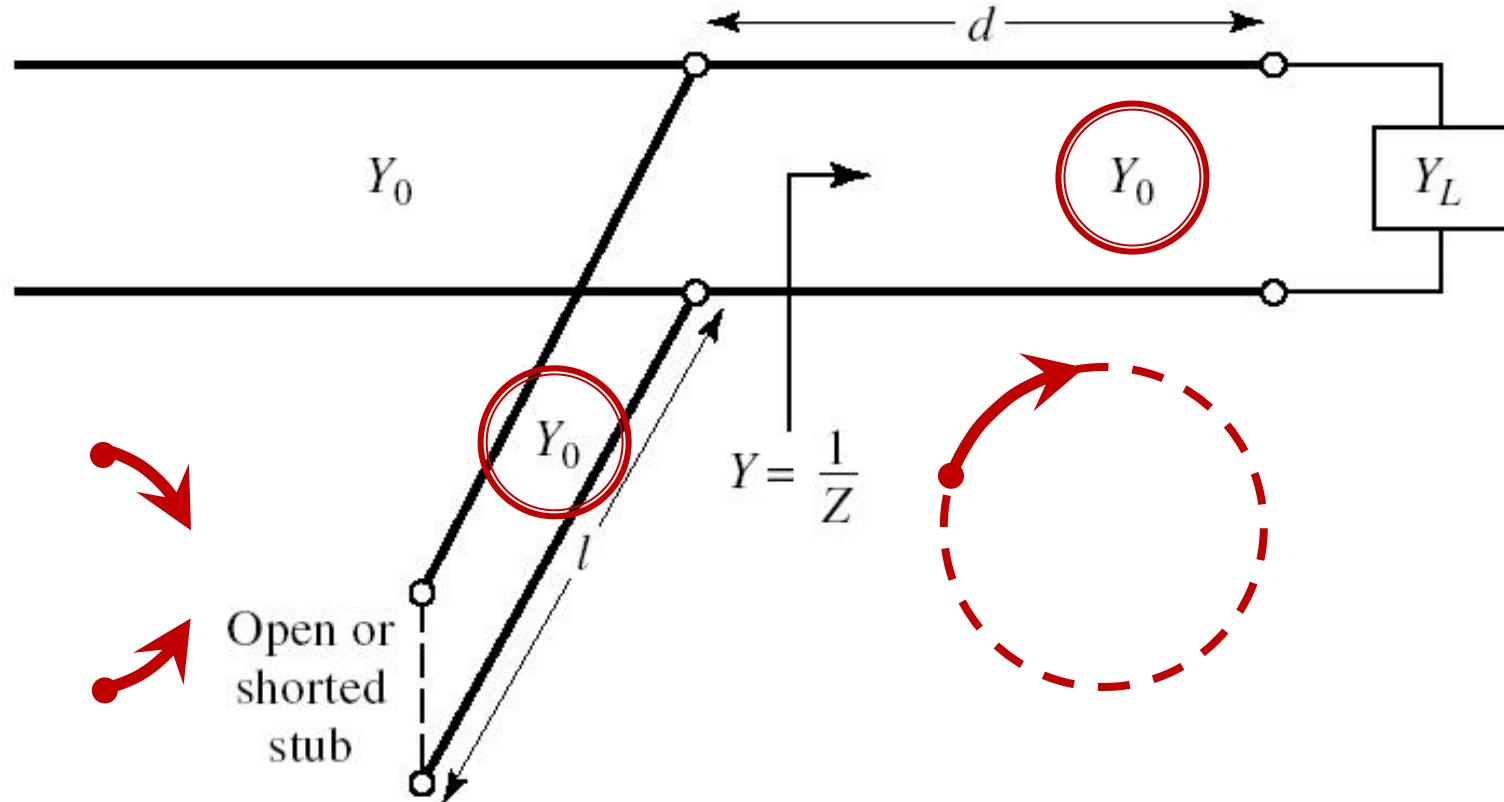
# **Impedance Matching with Stubs**

# Smith chart, $r=1$ and $g=1$



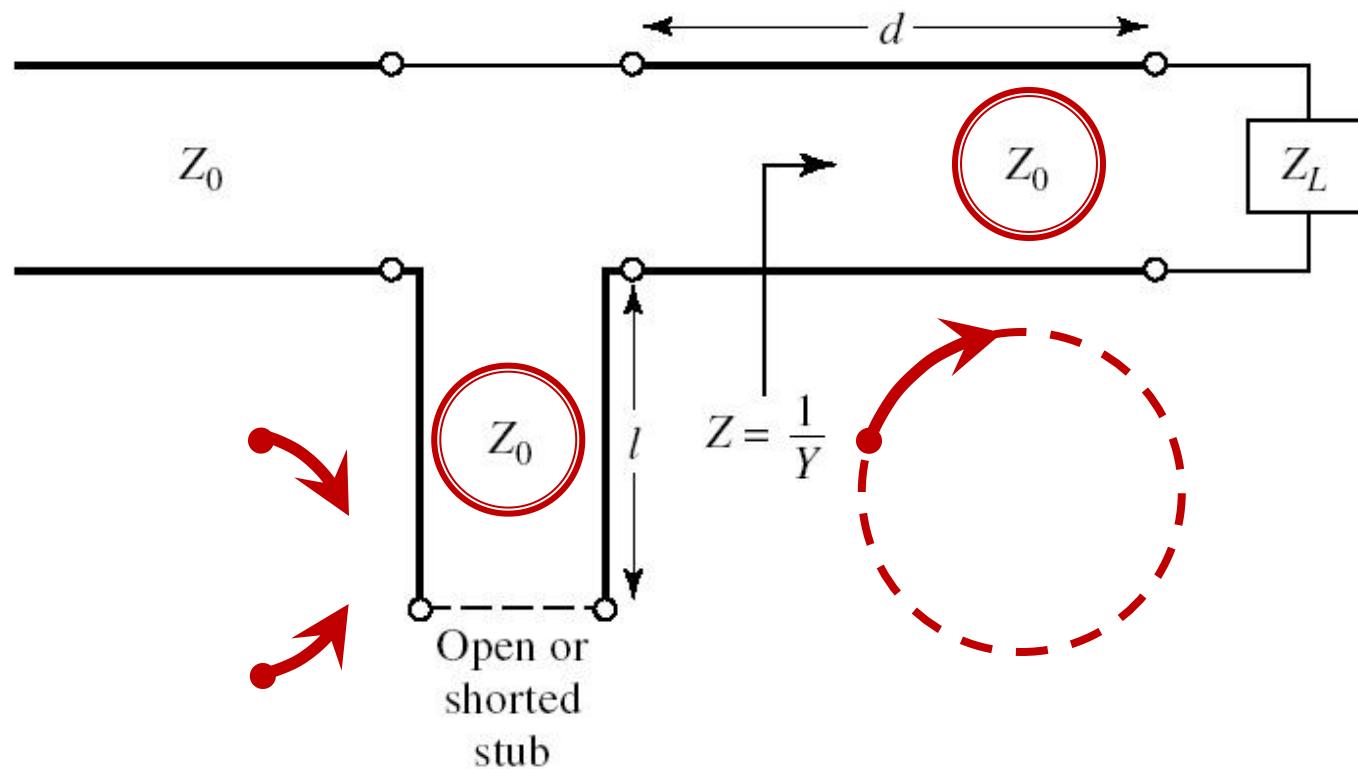
# Single stub tuning

- Shunt Stub



# Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)

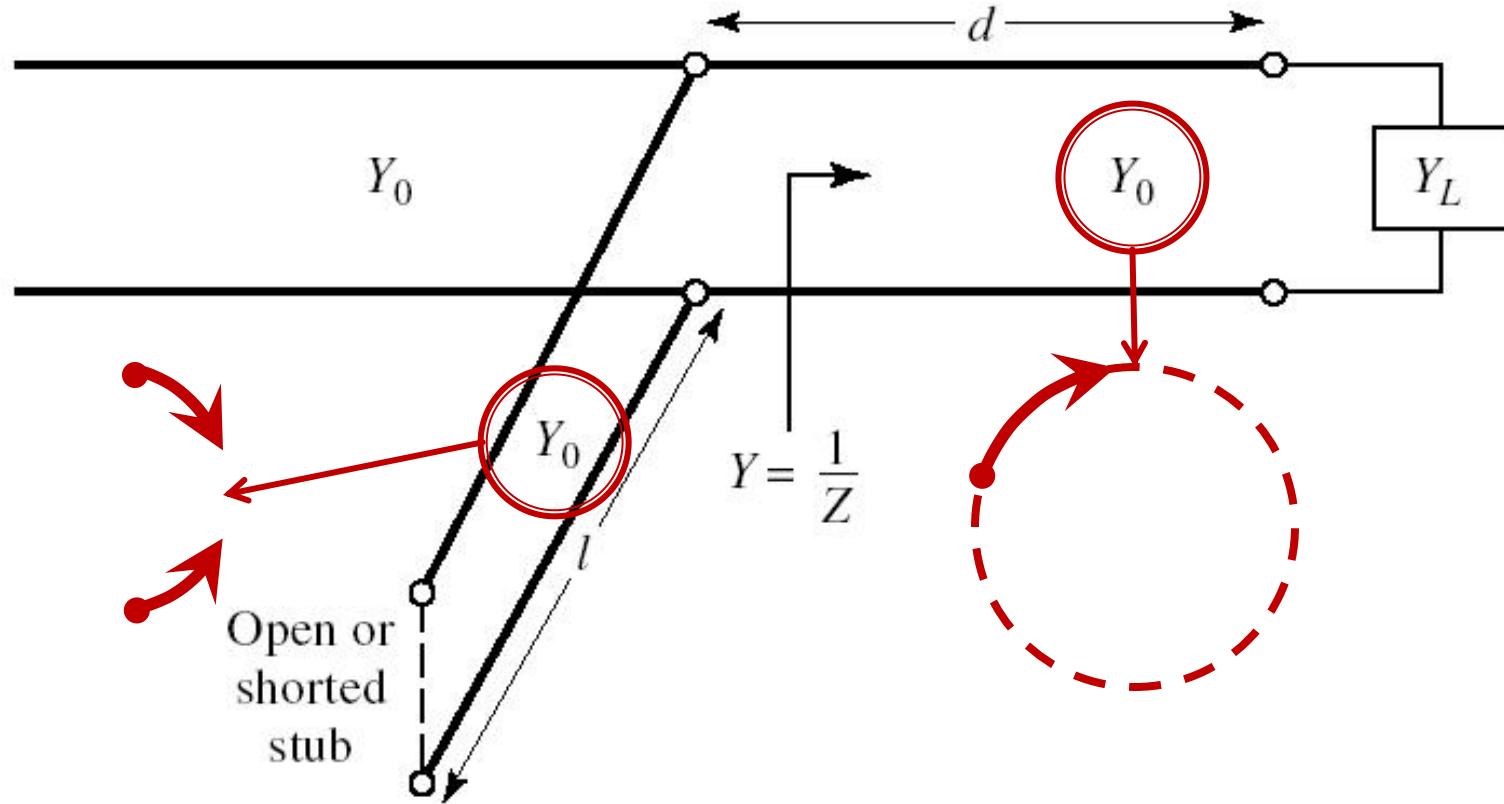


# **Shunt Stub**

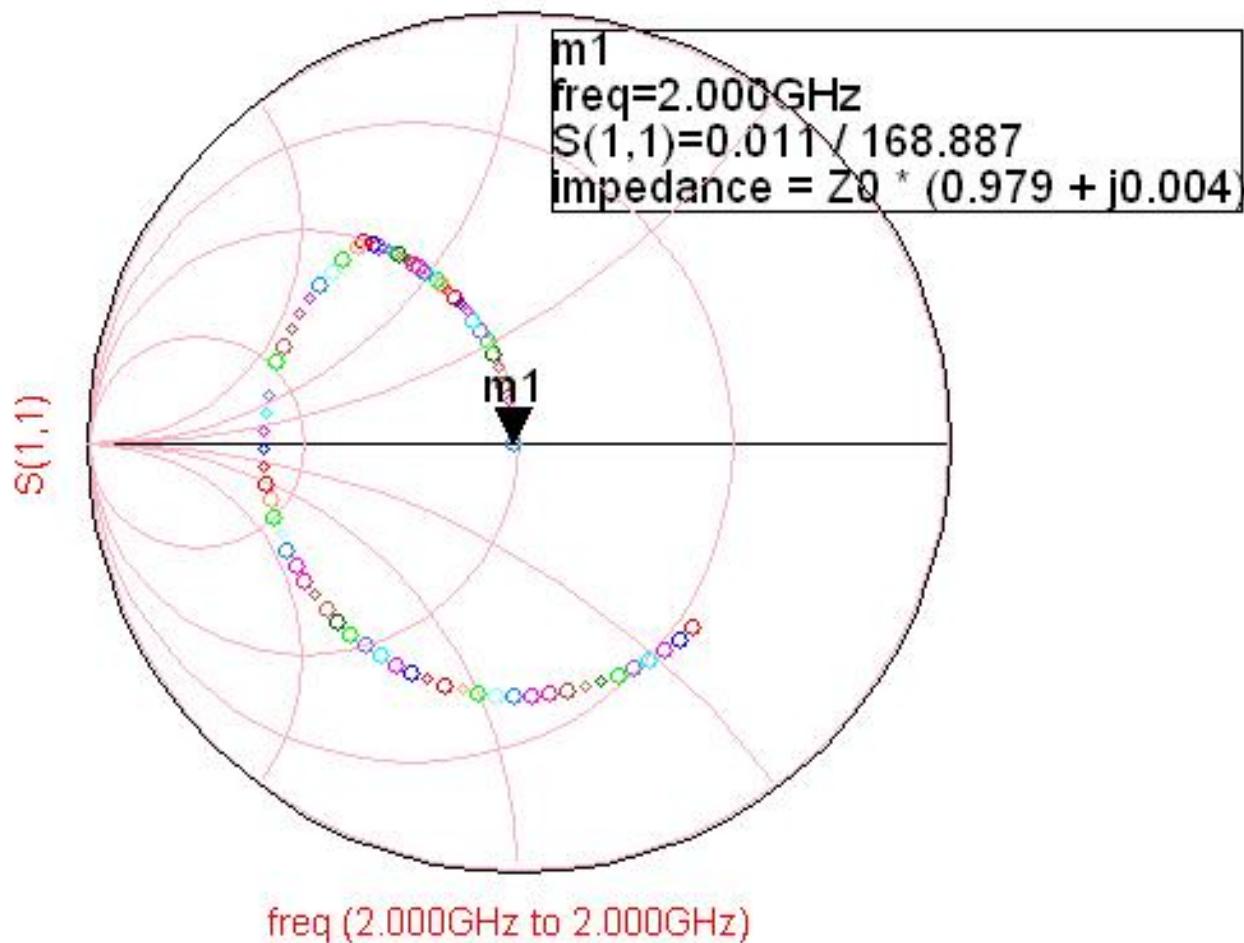
Sectiune de linie paralel

# Case 1, Shunt Stub

## Shunt Stub



# Example, Shunt Stub, sc.



# Shunt Stub, some notes

- mathematical functions which offer the input impedance in a stub are periodic functions of  $l$ ,  $\tan/\cot$  based functions

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

- adding or subtracting

$$E = \beta \cdot l = \pi = 180^\circ \quad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbb{N}$$

doesn't change the result (full rotation around the Smith Chart – hence the  $0.5\lambda$  gradation of the circumference of the diagram)

# Shunt Stub, some notes

- adding or subtracting  $\lambda/4$  transforms the impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

$$\tan \beta \cdot \left( l + \frac{\lambda}{4} \right) = \tan \left( \beta \cdot l + \frac{\pi}{2} \right) = \frac{\sin(\beta \cdot l + \pi/2)}{\cos(\beta \cdot l + \pi/2)} = \frac{\cos \beta \cdot l}{-\sin \beta \cdot l} = -\cot \beta \cdot l$$

from the open-circuited value to the short-circuited one and vice versa

- For tuning in ADS it's better to start from the neutral point (value of the electrical length of the line which doesn't influence the circuit)

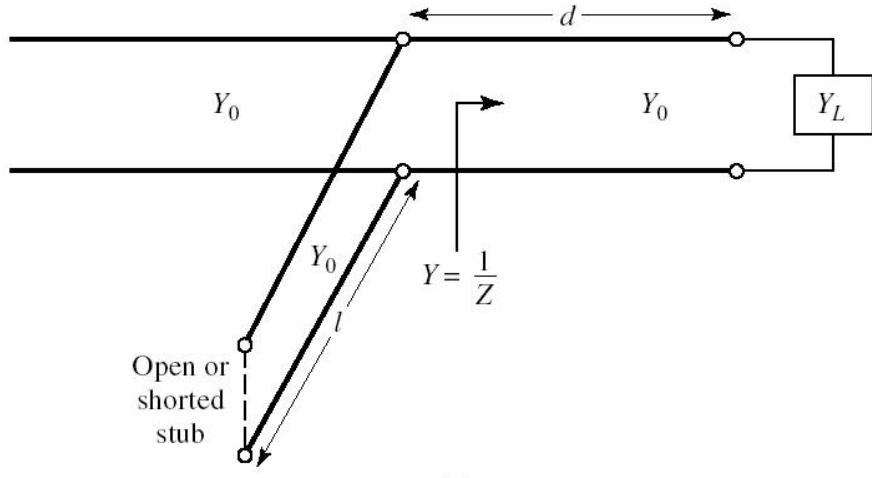
  - series line:  $E = \beta \cdot l = 0$

  - shunt stub:  $Z_{in} \rightarrow \infty, \tan \beta \cdot l / \cot \beta \cdot l \rightarrow \infty, E = 90^\circ / 0^\circ$

# Analytical solution

Shunt Stub

# Analytical solution, impedances



$$Z_L = \frac{1}{Y_L} = R_L + j \cdot X_L$$

$$Z = Z_0 \cdot \frac{(R_L + j \cdot X_L) + j \cdot Z_0 \cdot t}{Z_0 + j \cdot (R_L + j \cdot X_L) \cdot t}$$

$$t = \tan \beta \cdot d \quad Y = G + j \cdot B = \frac{1}{Z}$$

$$G = \frac{R_L \cdot (1 + t^2)}{R_L^2 + (X_L + Z_0 \cdot t)^2}$$

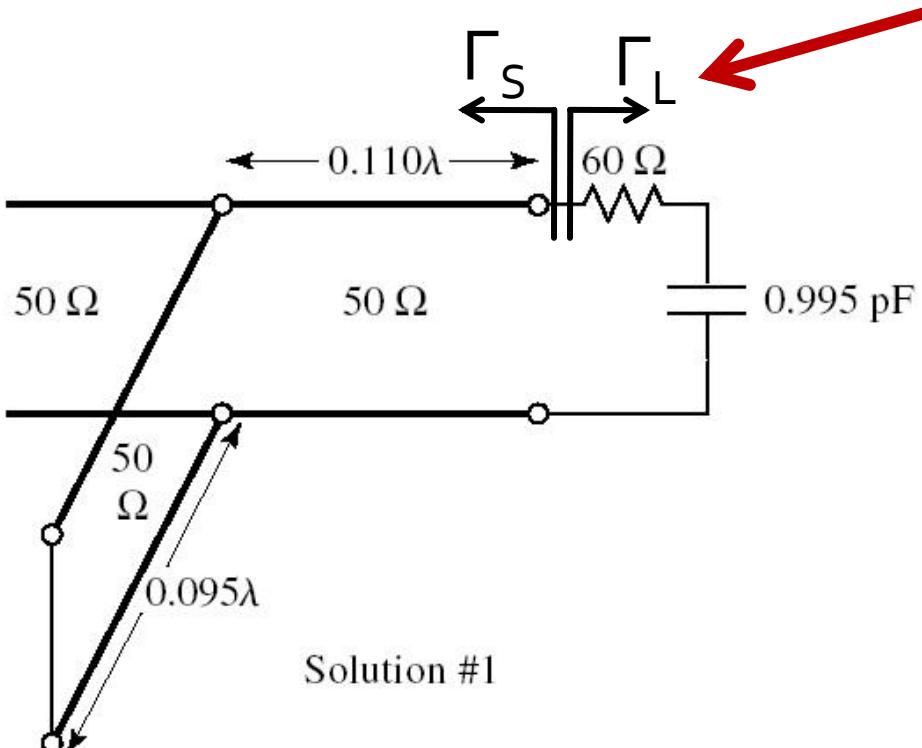
$$B = \frac{R_L^2 \cdot t - (Z_0 - X_L \cdot t) \cdot (X_L + Z_0 \cdot t)}{Z_0 \cdot [R_L^2 + (X_L + Z_0 \cdot t)^2]}$$

- $d$  (which implies  $t$ ) is chosen so that:  $G = Y_0 = \frac{1}{Z_0}$

$$Z_0 \cdot (R_L - Z_0) \cdot t^2 - 2 \cdot X_L \cdot Z_0 \cdot t + (R_L \cdot Z_0 - R_L^2 - X_L^2) = 0$$

# Analytical solution, reflection coefficient

- load:  $60\Omega$  series with  $0.995\text{ pF}$  at  $2\text{GHz}$



$$Z_L = R_L + \frac{1}{j \cdot \omega \cdot C_L} = 60\Omega - j \cdot 79.977\Omega$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.405 - j \cdot 0.432$$

$$Y_L = \frac{1}{Z_L} = 0.006S + j \cdot 0.008S$$

$$y_L = \frac{Y_L}{Y_0} = 0.3 + j \cdot 0.4$$

- matching requires obtaining conjugate value for  $\Gamma$

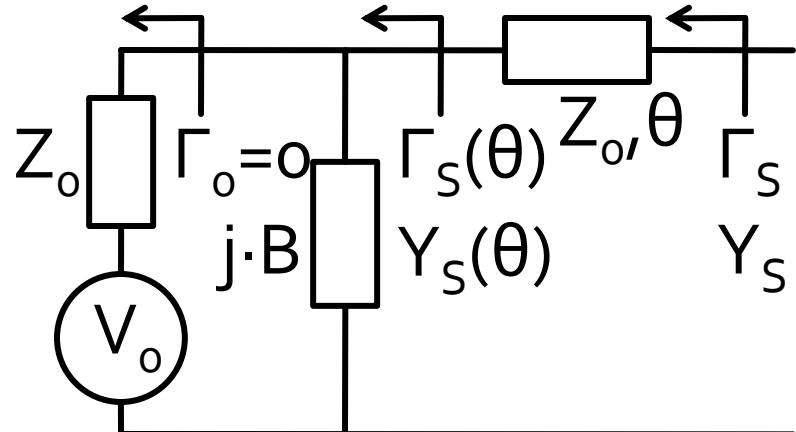
$$\Gamma_S = \Gamma_L^* = 0.405 + j \cdot 0.432$$

$$\Gamma_S = 0.593 \angle 46.85^\circ$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85^\circ$$

# Analytical solution, $\Gamma$

- series line
  - electrical length  $E = \beta \cdot l = \theta$
  - moves the reflection coefficient on the circle  $g=1$
- shunt stub
  - electrical length  $E = \beta \cdot l_{sp} = \theta_{sp}$
  - moves the reflection coefficient to the center of the Smith Chart ( $\Gamma_o=0$ )



$$y_s = \frac{Y_s}{Y_0} = Y_s \cdot Z_0 = Y_s \cdot 50\Omega$$

$$y_s = \frac{1 - \Gamma_s}{1 + \Gamma_s} = 0.3 - j \cdot 0.4$$

$$\Gamma_s(\theta) = [\Gamma_L(\theta)]^* = [\Gamma_L \cdot e^{-2j\theta}]^*$$

$$\Gamma_s(\theta) = \Gamma_L^* \cdot e^{2j\theta} = \Gamma_s \cdot e^{2j\theta}$$

$$y_s(\theta) = \frac{1 - \Gamma_s(\theta)}{1 + \Gamma_s(\theta)} = \frac{1 - \Gamma_s \cdot e^{2j\theta}}{1 + \Gamma_s \cdot e^{2j\theta}}$$

# A.S., $\Gamma$ , series line, proof

- After the series line with electrical length  $\theta$ :

$$\operatorname{Re}[y_S(\theta)] = 1$$

$$\operatorname{Im}[y_S(\theta)] = B$$

$$\operatorname{Re}[y_S(\theta)] = \frac{1}{2} \cdot [y_S(\theta) + y_S^*(\theta)]$$

$$\operatorname{Im}[y_S(\theta)] = \frac{1}{2j} \cdot [y_S(\theta) - y_S^*(\theta)]$$

$$\operatorname{Re}[y_S(\theta)] = \frac{1}{2} \cdot \left[ \frac{1 - \Gamma_S \cdot e^{2j\theta}}{1 + \Gamma_S \cdot e^{2j\theta}} + \frac{1 - \Gamma_S^* \cdot e^{-2j\theta}}{1 + \Gamma_S^* \cdot e^{-2j\theta}} \right] \quad \Gamma_S = |\Gamma_S| \cdot e^{j\varphi}$$

$$\operatorname{Re}[y_S(\theta)] = \frac{1}{2} \cdot \left[ \frac{(1 - |\Gamma_S| \cdot e^{j(\varphi+2\theta)}) \cdot (1 + |\Gamma_S| \cdot e^{-j(\varphi+2\theta)}) + (1 - |\Gamma_S| \cdot e^{-j(\varphi+2\theta)}) \cdot (1 + |\Gamma_S| \cdot e^{j(\varphi+2\theta)})}{(1 + |\Gamma_S| \cdot e^{-j(\varphi+2\theta)}) \cdot (1 + |\Gamma_S| \cdot e^{j(\varphi+2\theta)})} \right]$$

$$\operatorname{Re}[y_S(\theta)] = \frac{1}{2} \cdot \left[ \frac{2 - 2 \cdot |\Gamma_S|^2}{1 + |\Gamma_S|^2 + 2 \cdot |\Gamma_S| \cdot \cos(\varphi + 2\theta)} \right] = 1 \Rightarrow \boxed{\cos(\varphi + 2\theta) = -|\Gamma_S|}$$

# A.S., $\Gamma$ , series line, usage

- Equations for computing the series line  $\theta$ :

$$\operatorname{Re}[y_s(\theta)] = 1 \Rightarrow \boxed{\cos(\varphi + 2\theta) = -|\Gamma_s|}$$

$$\Gamma_s = |\Gamma_s| \cdot e^{j\varphi} \quad \Gamma_s = 0.593 \angle 46.85^\circ \quad |\Gamma_s| = 0.593; \quad \varphi = 46.85^\circ$$

- two solutions possible, in the  $0 \div 180^\circ$  range

- add  $\lambda/2 \Leftrightarrow 180^\circ$  as needed

$$\theta = \frac{1}{2} \cdot [\pm \cos^{-1}(-|\Gamma_s|) - \varphi + k \cdot 360^\circ] = \frac{1}{2} \cdot [\pm \cos^{-1}(-|\Gamma_s|) - \varphi] + k \cdot 180^\circ$$

$\forall k \in N$

$$\cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35^\circ$$

$$(46.85^\circ + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases}$$

$$\theta = \begin{cases} +39.7^\circ \\ -86.6^\circ + 180^\circ = +93.4^\circ \end{cases}$$

# A.S., $\Gamma$ , shunt stub, proof

- Equations for computing the shunt stub  $\theta_{sp}$ :

$$\operatorname{Re}[y_s(\theta)] = 1 \quad \cos(\varphi + 2\theta) = -|\Gamma_s|$$

$$\operatorname{Im}[y_s(\theta)] = \frac{1}{2j} \cdot \left[ \frac{1 - \Gamma_s \cdot e^{2j\theta}}{1 + \Gamma_s \cdot e^{2j\theta}} - \frac{1 - \Gamma_s^* \cdot e^{-2j\theta}}{1 + \Gamma_s^* \cdot e^{-2j\theta}} \right] \quad \Gamma_s = |\Gamma_s| \cdot e^{j\varphi}$$

$$\operatorname{Im}[y_s(\theta)] = \frac{1}{2j} \cdot \left[ \frac{(1 - |\Gamma_s| \cdot e^{j(\varphi+2\theta)}) \cdot (1 + |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) - (1 - |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) \cdot (1 + |\Gamma_s| \cdot e^{j(\varphi+2\theta)})}{(1 + |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) \cdot (1 + |\Gamma_s| \cdot e^{j(\varphi+2\theta)})} \right]$$

$$\operatorname{Im}[y_s(\theta)] = \frac{1}{2j} \cdot \left[ \frac{2 \cdot |\Gamma_s| \cdot e^{-j(\varphi+2\theta)} - 2 \cdot |\Gamma_s| \cdot e^{+j(\varphi+2\theta)}}{1 + |\Gamma_s|^2 + 2 \cdot |\Gamma_s| \cdot \cos(\varphi + 2\theta)} \right] = \frac{-2 \cdot |\Gamma_s| \cdot \sin(\varphi + 2\theta)}{1 + |\Gamma_s|^2 + 2 \cdot |\Gamma_s| \cdot \cos(\varphi + 2\theta)}$$

$$\cos(\varphi + 2\theta) = -|\Gamma_s| \Rightarrow \operatorname{Im}[y_s(\theta)] = \frac{-2 \cdot |\Gamma_s| \cdot \sin(\varphi + 2\theta)}{1 - |\Gamma_s|^2}$$

# A.S., $\Gamma$ , shunt stub, proof

- Equations for computing the shunt stub

$$\cos(\varphi + 2\theta) = -|\Gamma_s| \Rightarrow \sin(\varphi + 2\theta) = \pm \sqrt{1 - |\Gamma_s|^2}$$
$$\text{Im}[y_s(\theta)] = \frac{-2 \cdot |\Gamma_s| \cdot \sin(\varphi + 2\theta)}{1 - |\Gamma_s|^2} \Rightarrow \text{Im}[y_s(\theta)] = \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

- two cases

$$\varphi + 2\theta \in [0^\circ, 180^\circ] \Rightarrow \sin(\varphi + 2\theta) \geq 0$$

$$\begin{cases} \sin(\varphi + 2\theta) = \sqrt{1 - |\Gamma_s|^2} \\ \text{Im}[y_s(\theta)] = \frac{-2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} \end{cases}$$

$$\varphi + 2\theta \in (-180^\circ, 0^\circ) \Rightarrow \sin(\varphi + 2\theta) < 0$$
$$\begin{cases} \sin(\varphi + 2\theta) = -\sqrt{1 - |\Gamma_s|^2} \\ \text{Im}[y_s(\theta)] = \frac{2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} \end{cases}$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

# A.S., $\Gamma$ , shunt stub, proof

- We prefer (for microstrip) open circuited stub

$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

- The normalized susceptance to be introduced to achieve the match
  - $Y(\theta)$  is the admittance seen **towards** the source,  $Z_0$  parallel with  $j \cdot B$

$$b = \text{Im} \left[ \frac{Y_{in,oc}}{Y_0} \right] = \text{Im} \left[ \frac{Z_0}{Z_{in,oc}} \right] = \tan \beta \cdot l = \text{Im}[y_s(\theta)]$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

# Analytical solution, $\Gamma$ , usage

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \text{Im}[y_s(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

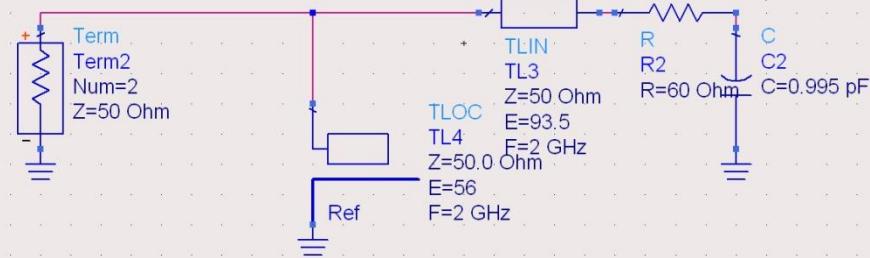
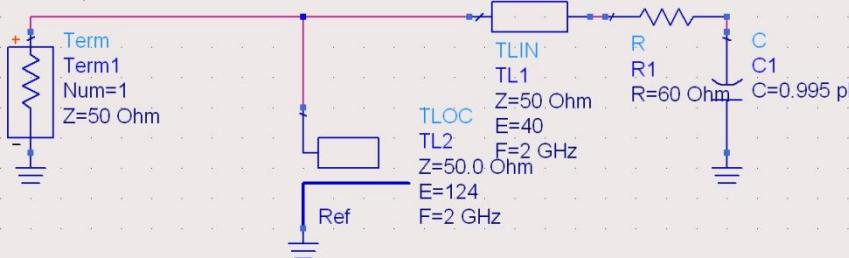
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$

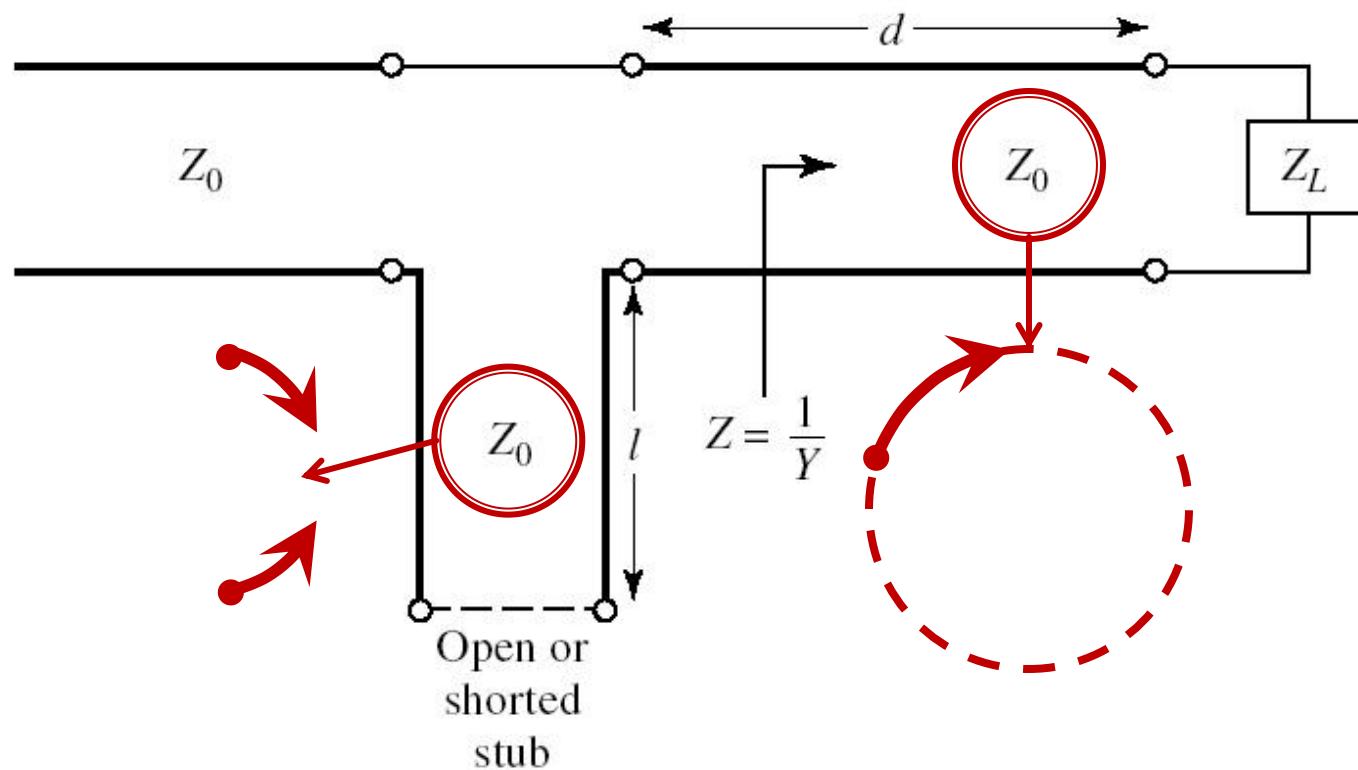


# **Series Stub**

Sectiune de linie serie

# Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



# Case 2, Series Stub

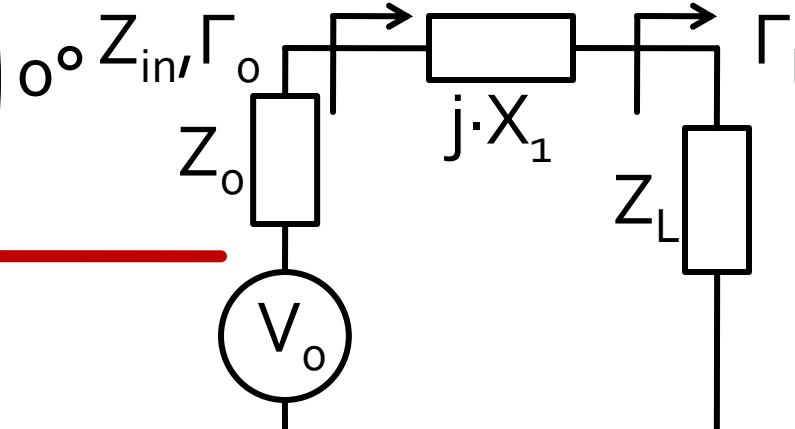
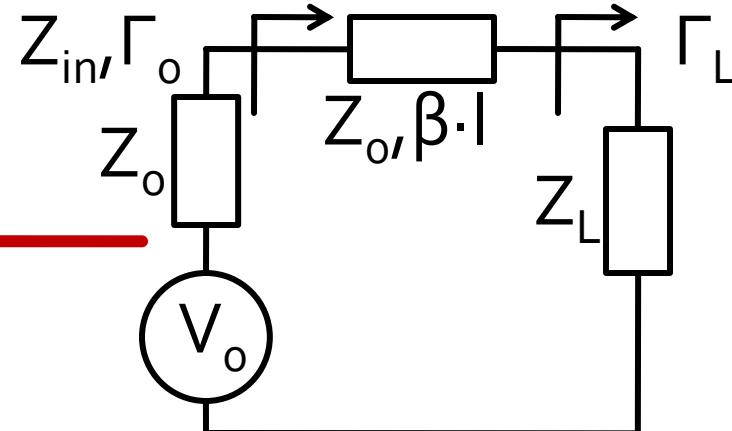
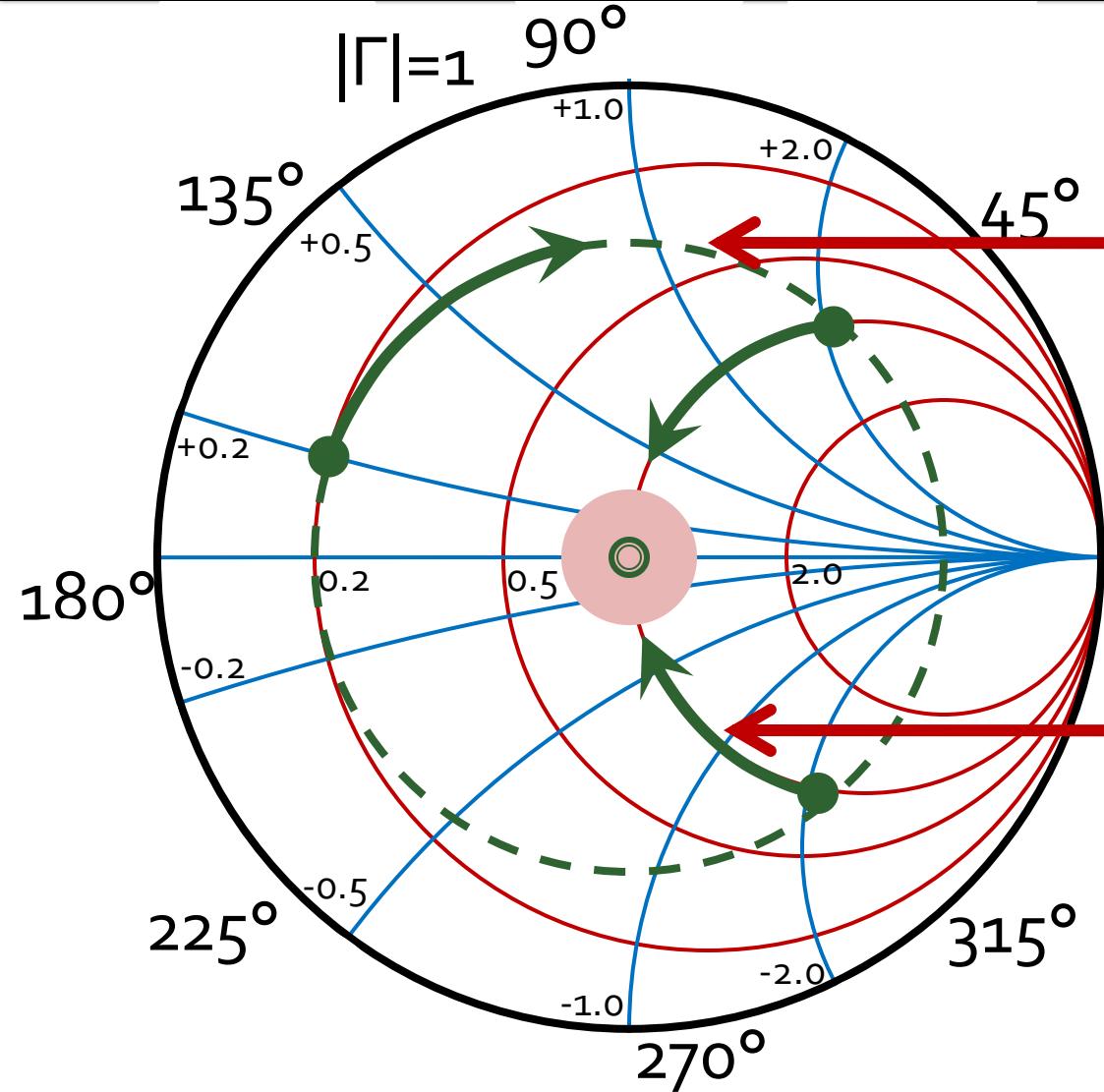
- We use a series transmission line to move the reflection coefficient **on the circle**  $r_L = 1$
- We compensate the remaining reactive part of the load with a series reactance to achieve match
- The series reactance is made with a stub which can be, as needed:
  - open-circuited
  - short-circuited

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

# Matching, series line + series reactance

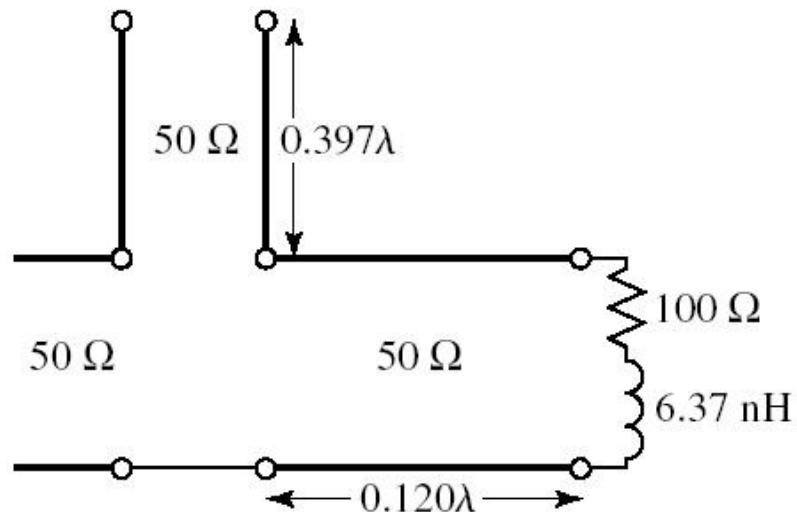


$$|\Gamma_{in}| = |\Gamma_L|$$

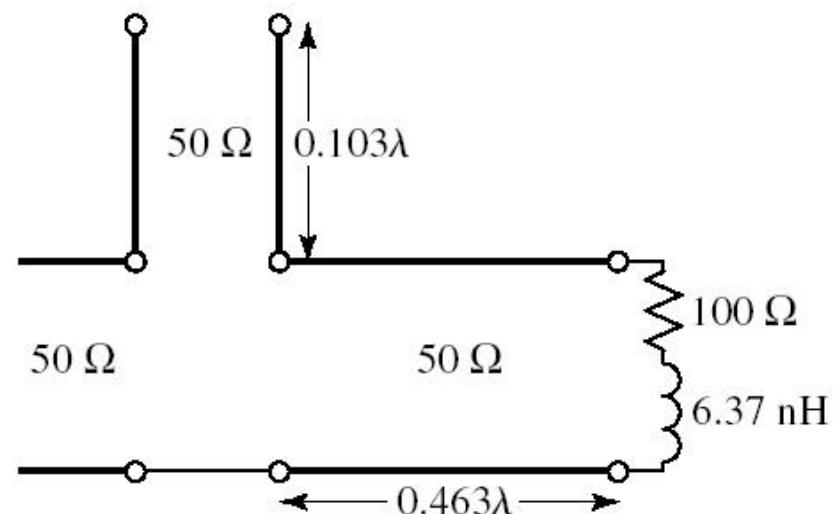
$$r_{in} = 1$$

# Example, Series Stub, oc.

- load:  $100 \Omega$  series with  $6.37 \text{ nH}$  at  $2 \text{ GHz}$
- two solutions possible

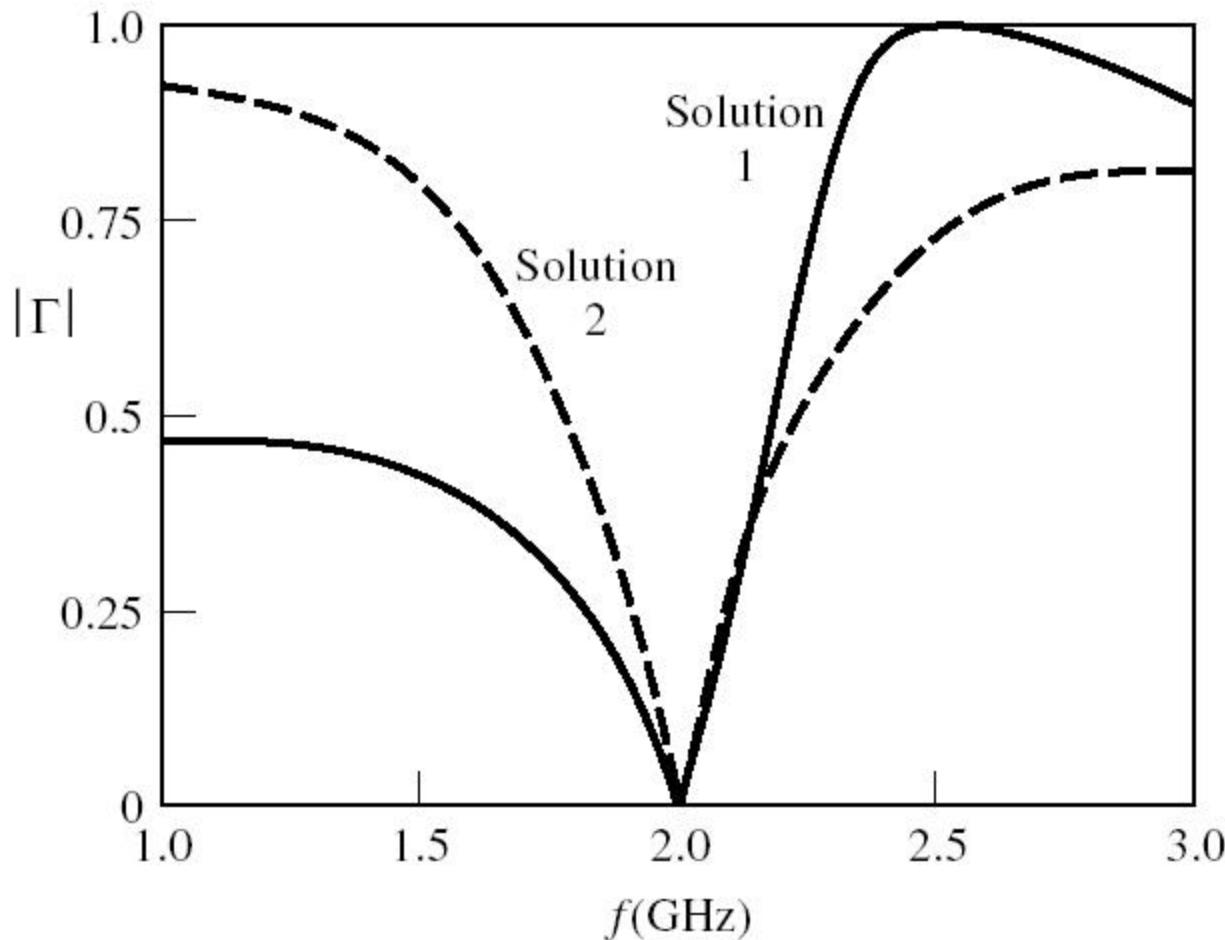


Solution 1

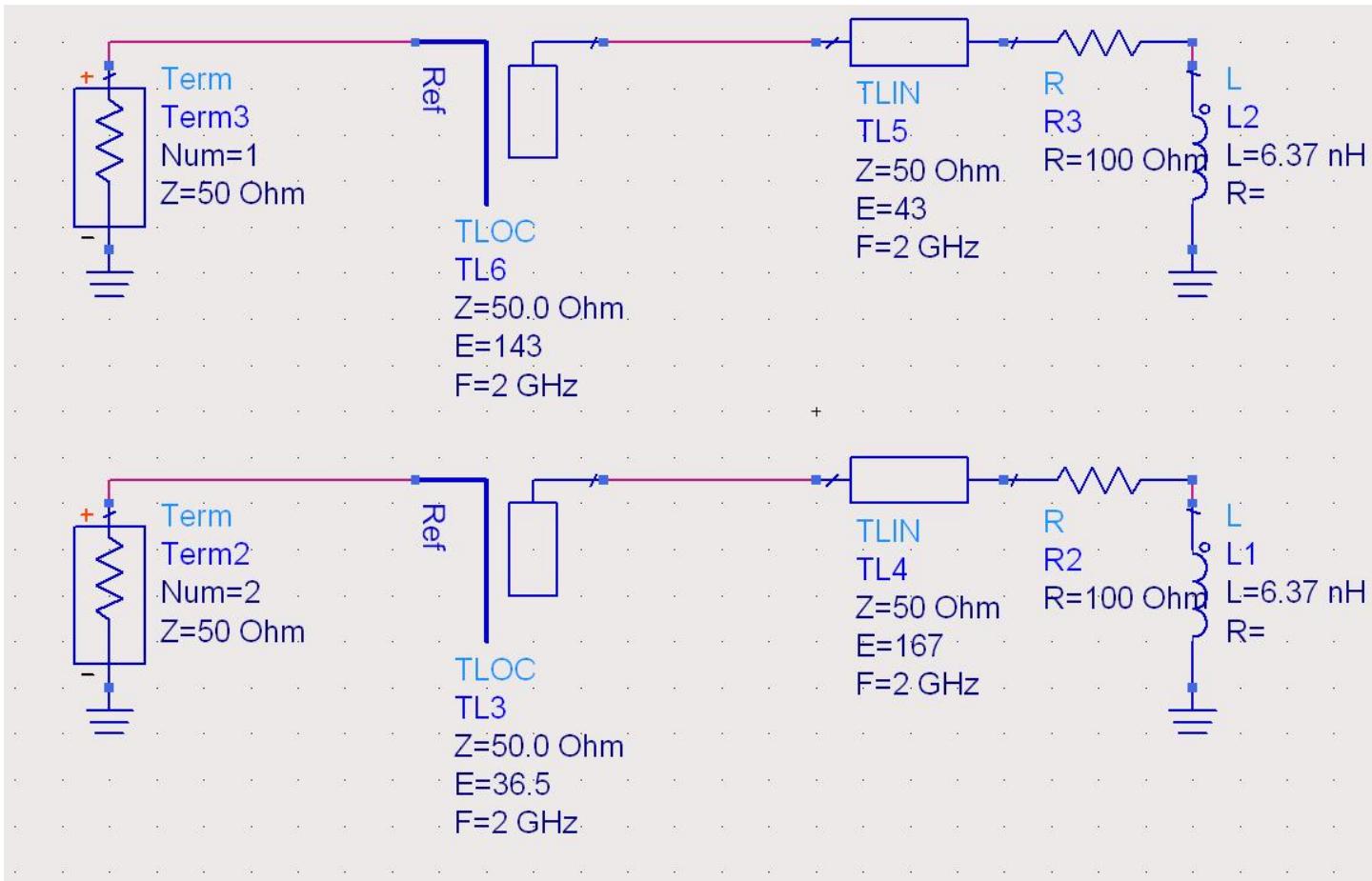


Solution 2

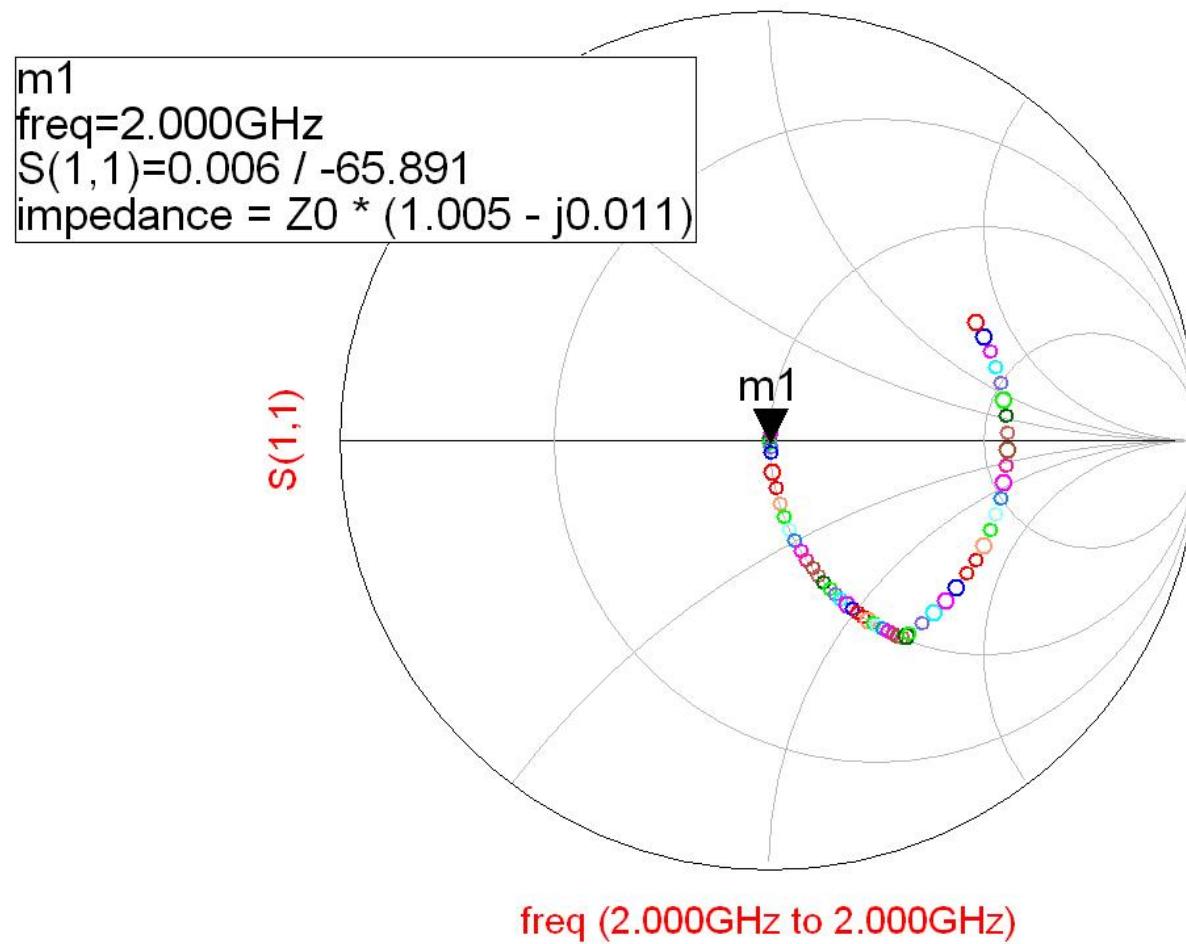
# Example, Series Stub, oc.



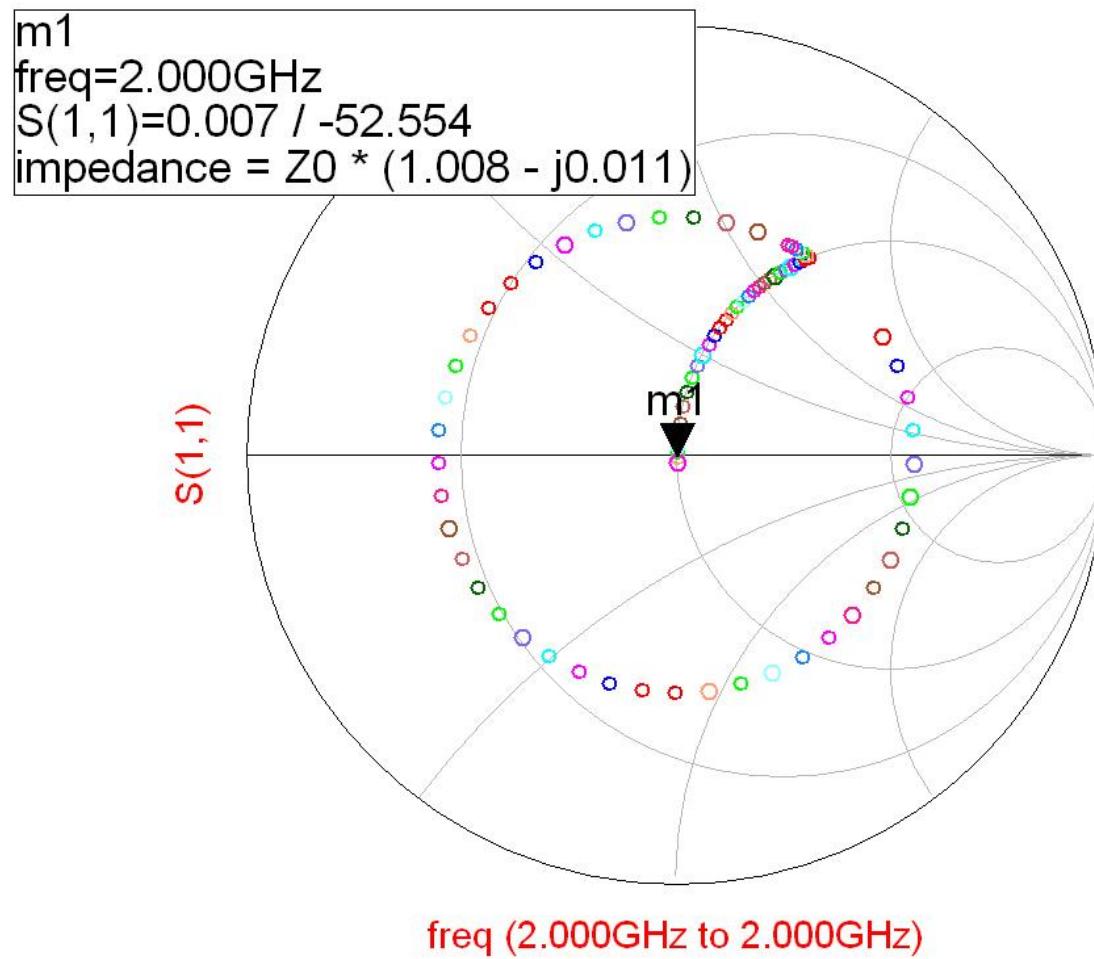
# Example, Series Stub, oc.



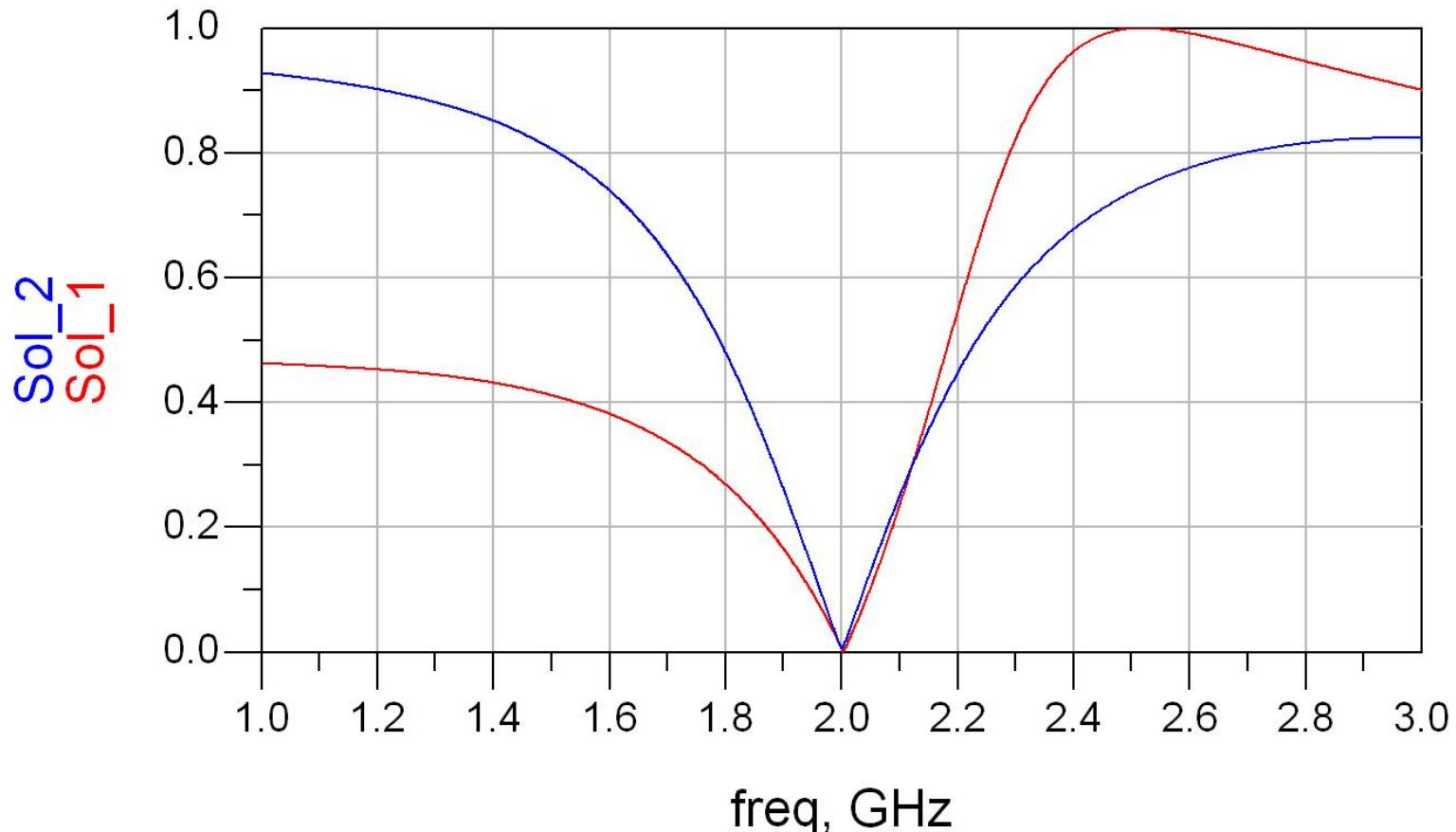
# Example, Series Stub, oc.



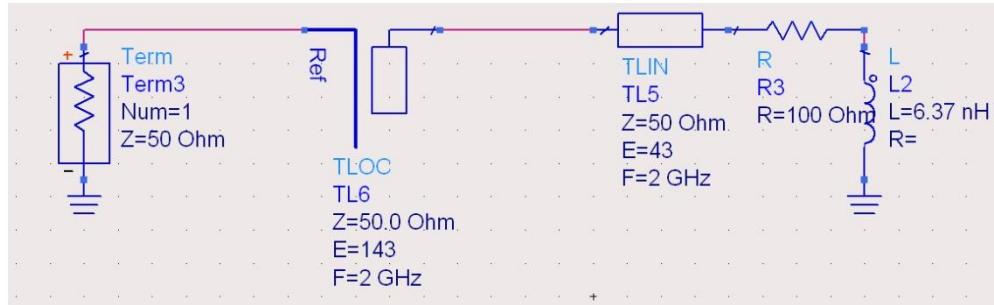
# Example, Series Stub, oc.



# Example, Series Stub, oc.



# Example, Series Stub, oc.

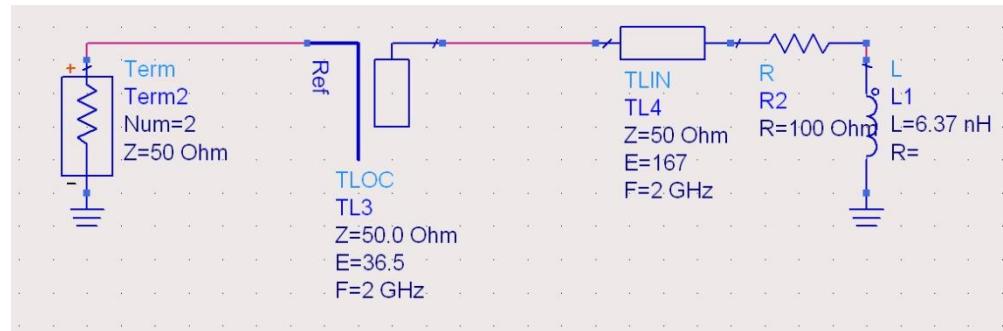


$$l_1 = \frac{43^\circ}{360^\circ} \cdot \lambda = 0.119 \cdot \lambda$$

$$l_2 = \frac{143^\circ}{360^\circ} \cdot \lambda = 0.397 \cdot \lambda$$

$$l_1 = \frac{167^\circ}{360^\circ} \cdot \lambda = 0.464 \cdot \lambda$$

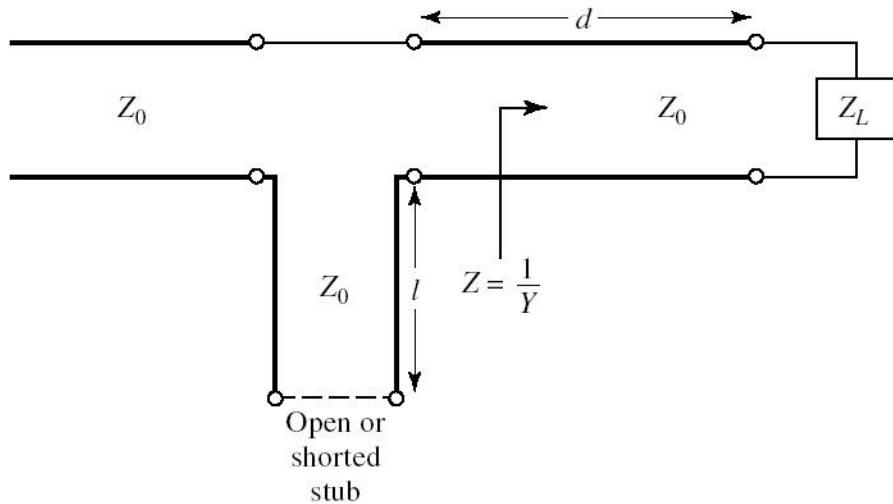
$$l_2 = \frac{36.5^\circ}{360^\circ} \cdot \lambda = 0.101 \cdot \lambda$$



# Analytical solution

Series Stub

# Analytical solution, impedances



$$Y_L = \frac{1}{Z_L} = G_L + j \cdot B_L$$

$$Y = Y_0 \cdot \frac{(G_L + j \cdot B_L) + j \cdot Y_0 \cdot t}{Y_0 + j \cdot (G_L + j \cdot B_L) \cdot t}$$

$$t = \tan \beta \cdot d \quad \text{not} \quad Z = R + j \cdot X = \frac{1}{Y}$$

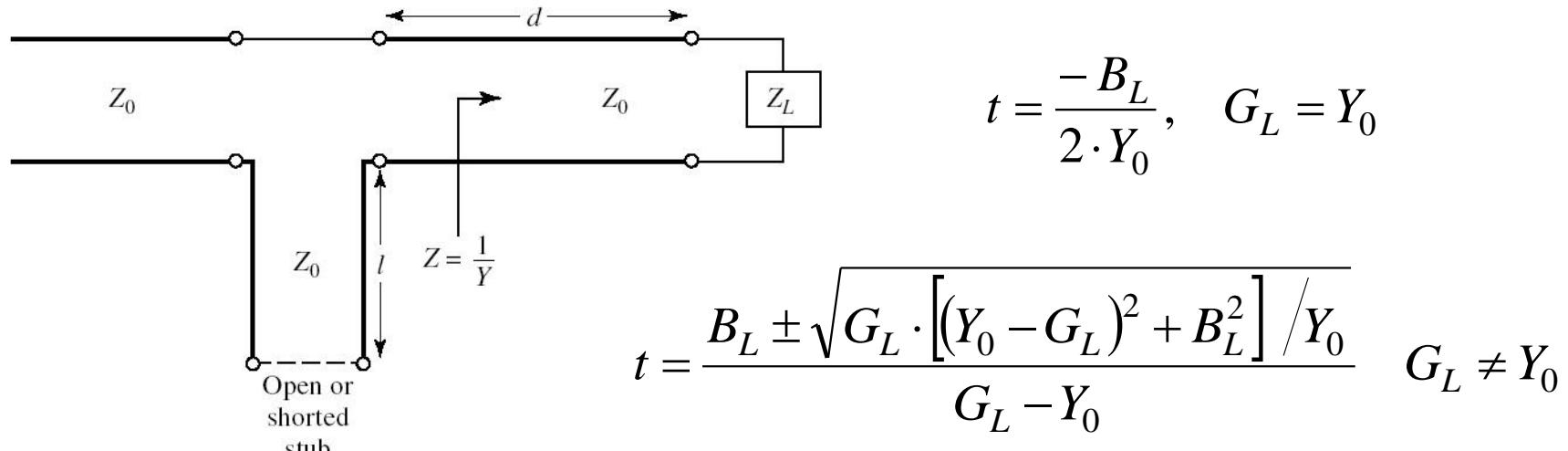
$$R = \frac{G_L \cdot (1 + t^2)}{G_L^2 + (G_L + Y_0 \cdot t)^2}$$

$$X = \frac{G_L^2 \cdot t - (Y_0 - B_L \cdot t) \cdot (B_L + Y_0 \cdot t)}{Y_0 \cdot [G_L^2 + (B_L + Y_0 \cdot t)^2]}$$

- $d$  (which implies  $t$ ) is chosen so that:  $R = Z_0 = \frac{1}{Y_0}$

$$Y_0 \cdot (G_L - Y_0) \cdot t^2 - 2 \cdot B_L \cdot Y_0 \cdot t + (G_L \cdot Y_0 - G_L^2 - B_L^2) = 0$$

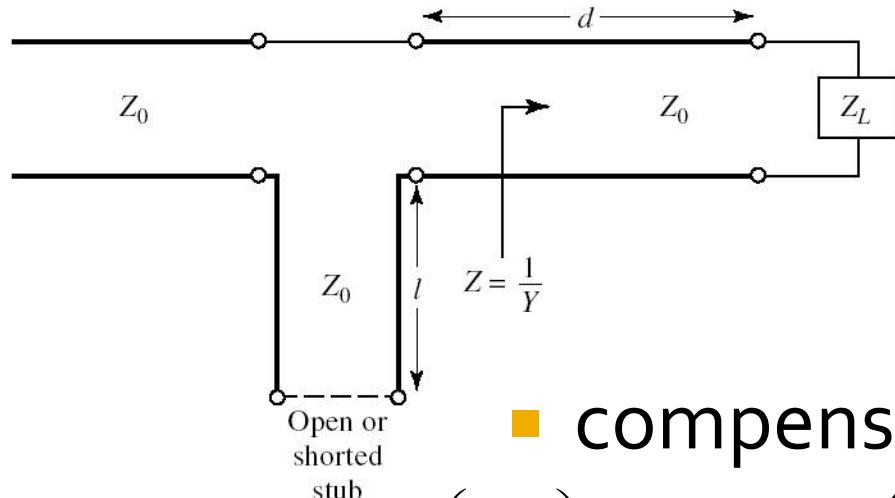
# Analytical solution



- second grade equation, 2 solutions possible
- $d$  computed from  $t$

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi} \cdot \arctan t & t \geq 0 \\ \frac{1}{2\pi} \cdot (\pi + \arctan t) & t < 0 \end{cases}$$

# Analytical solution



$$X_S = -X$$

$$X = \frac{G_L^2 \cdot t - (Y_0 - B_L \cdot t) \cdot (B_L + Y_0 \cdot t)}{Y_0 \cdot [G_L^2 + (B_L + Y_0 \cdot t)^2]}$$

■ compensating reactance is:

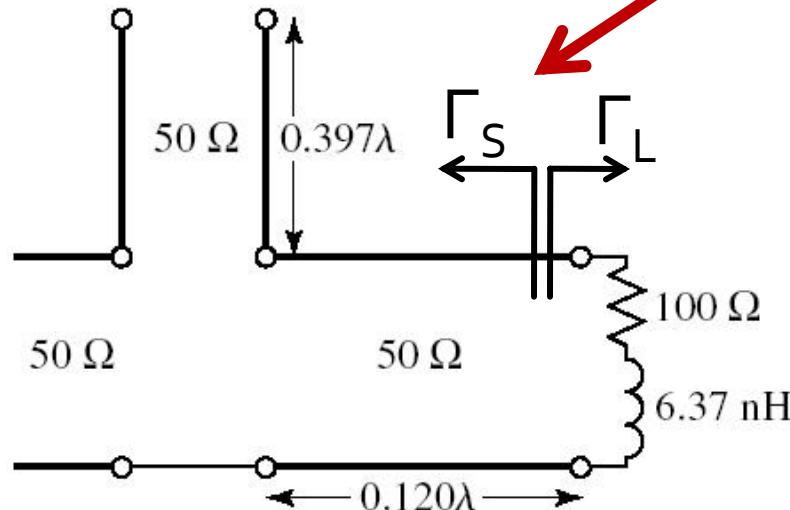
$$\frac{l_{sc}}{\lambda} = \frac{1}{2\pi} \cdot \arctan \left( \frac{X_S}{Z_0} \right) = \frac{-1}{2\pi} \cdot \arctan \left( \frac{X}{Z_0} \right)$$

$$\frac{l_{oc}}{\lambda} = \frac{-1}{2\pi} \cdot \arctan \left( \frac{Z_0}{X_S} \right) = \frac{1}{2\pi} \cdot \arctan \left( \frac{Z_0}{X} \right)$$

■ for **negative lengths** we add  $\lambda/2$

# Analytical solution, reflection coefficient

- load:  $100 \Omega$  series with  $6.37 \text{ nH}$  at  $2 \text{ GHz}$



Solution 1

$$Z_L = R_L + \frac{1}{j \cdot \omega \cdot C_L} = 100\Omega + j \cdot 80.05\Omega$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.481 + j \cdot 0.277$$

$$z_L = \frac{Z_L}{Z_0} = 2 + j \cdot 1.6$$

- matching requires obtaining conjugate value for  $\Gamma$

$$\Gamma_s = \Gamma_L^* = 0.481 - j \cdot 0.277$$

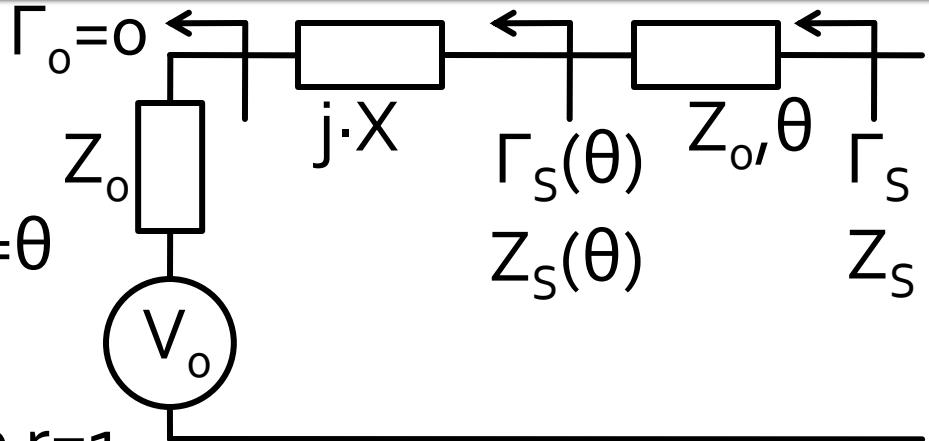
$$\Gamma_s = 0.555 \angle -29.92^\circ$$

$$|\Gamma_s| = 0.555; \quad \varphi = -29.92^\circ$$

# Analytical solution, $\Gamma$

## ■ series line

- electrical length  $E = \beta \cdot l = \theta$
- moves the reflection coefficient on the circle  $r=1$



## ■ series stub

- electrical length  $E = \beta \cdot l_{ss} = \theta_{ss}$
- moves the reflection coefficient to the center of the Smith Chart ( $\Gamma_o = 0$ )

$$z_s = \frac{Z_s}{Z_0} = \frac{Z_s}{50\Omega}$$

$$z_s = \frac{1 + \Gamma_s}{1 - \Gamma_s} = 2 - j \cdot 1.6$$

$$\Gamma_s(\theta) = \Gamma_s \cdot e^{2j\theta}$$

$$z_s(\theta) = \frac{1 + \Gamma_s(\theta)}{1 - \Gamma_s(\theta)} = \frac{1 + \Gamma_s \cdot e^{2j\theta}}{1 - \Gamma_s \cdot e^{2j\theta}}$$

# A.S., $\Gamma$ , series line, proof

- After the series line with electrical length  $\theta$ :

$$\operatorname{Re}[z_s(\theta)] = 1 \quad \operatorname{Im}[z_s(\theta)] = X$$

$$\operatorname{Re}[z_s(\theta)] = \frac{1}{2} \cdot [z_s(\theta) + z_s^*(\theta)] \quad \operatorname{Im}[z_s(\theta)] = \frac{1}{2j} \cdot [z_s(\theta) - z_s^*(\theta)]$$

$$\operatorname{Re}[z_s(\theta)] = \frac{1}{2} \cdot \left[ \frac{1 + \Gamma_s \cdot e^{2j\theta}}{1 - \Gamma_s \cdot e^{2j\theta}} + \frac{1 + \Gamma_s^* \cdot e^{-2j\theta}}{1 - \Gamma_s^* \cdot e^{-2j\theta}} \right] \quad \Gamma_s = |\Gamma_s| \cdot e^{j\varphi}$$

$$\operatorname{Re}[z_s(\theta)] = \frac{1}{2} \cdot \left[ \frac{(1 + |\Gamma_s| \cdot e^{j(\varphi+2\theta)}) \cdot (1 - |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) + (1 + |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) \cdot (1 - |\Gamma_s| \cdot e^{j(\varphi+2\theta)})}{(1 - |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) \cdot (1 - |\Gamma_s| \cdot e^{j(\varphi+2\theta)})} \right]$$

$$\operatorname{Re}[z_s(\theta)] = \frac{1}{2} \cdot \left[ \frac{2 - 2 \cdot |\Gamma_s|^2}{1 + |\Gamma_s|^2 - 2 \cdot |\Gamma_s| \cdot \cos(\varphi + 2\theta)} \right] = 1 \Rightarrow \boxed{\cos(\varphi + 2\theta) = |\Gamma_s|}$$

# A.S., $\Gamma$ , series line. usage

- Equations for computing the series line  $\theta$ :

$$\operatorname{Re}[z_s(\theta)] = 1 \Rightarrow \cos(\varphi + 2\theta) = |\Gamma_s|$$

$$\Gamma_s = |\Gamma_s| \cdot e^{j\varphi} \quad \Gamma_s = 0.555 \angle -29.92^\circ \quad |\Gamma_s| = 0.555; \quad \varphi = -29.92^\circ$$

- two solutions possible, in the  $0 \div 180^\circ$  range (add  $\lambda/2 \Leftrightarrow 180^\circ$  as needed)

$$\theta = \frac{1}{2} \cdot [\pm \cos^{-1}(|\Gamma_s|) - \varphi + k \cdot 360^\circ] = \frac{1}{2} \cdot [\pm \cos^{-1}(|\Gamma_s|) - \varphi] + k \cdot 180^\circ$$

$\forall k \in N$

$$\cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28^\circ$$

$$(-29.92^\circ + 2\theta) = \begin{cases} +56.28^\circ \\ -56.28^\circ \end{cases} \quad \theta = \begin{cases} +43.1^\circ \\ -13.2^\circ + 180^\circ = +166.8^\circ \end{cases}$$

# A.S., $\Gamma$ , series stub, proof

- Equations for computing the series stub  $\theta_{ss}$ :

$$\operatorname{Re}[z_s(\theta)] = 1 \quad \cos(\varphi + 2\theta) = |\Gamma_s|$$

$$\operatorname{Im}[z_s(\theta)] = \frac{1}{2j} \cdot \left[ \frac{1 + \Gamma_s \cdot e^{2j\theta}}{1 - \Gamma_s \cdot e^{2j\theta}} - \frac{1 + \Gamma_s^* \cdot e^{-2j\theta}}{1 - \Gamma_s^* \cdot e^{-2j\theta}} \right] \quad \Gamma_s = |\Gamma_s| \cdot e^{j\varphi}$$

$$\operatorname{Im}[z_s(\theta)] = \frac{1}{2j} \cdot \left[ \frac{(1 + |\Gamma_s| \cdot e^{j(\varphi+2\theta)}) \cdot (1 - |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) - (1 + |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) \cdot (1 - |\Gamma_s| \cdot e^{j(\varphi+2\theta)})}{(1 - |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}) \cdot (1 - |\Gamma_s| \cdot e^{j(\varphi+2\theta)})} \right]$$

$$\operatorname{Im}[z_s(\theta)] = \frac{1}{2j} \cdot \left[ \frac{2 \cdot |\Gamma_s| \cdot e^{+j(\varphi+2\theta)} - 2 \cdot |\Gamma_s| \cdot e^{-j(\varphi+2\theta)}}{1 + |\Gamma_s|^2 - 2 \cdot |\Gamma_s| \cdot \cos(\varphi + 2\theta)} \right] = \frac{2 \cdot |\Gamma_s| \cdot \sin(\varphi + 2\theta)}{1 + |\Gamma_s|^2 - 2 \cdot |\Gamma_s| \cdot \cos(\varphi + 2\theta)}$$

$$\cos(\varphi + 2\theta) = |\Gamma_s| \Rightarrow \quad \operatorname{Im}[z_s(\theta)] = \frac{2 \cdot |\Gamma_s| \cdot \sin(\varphi + 2\theta)}{1 - |\Gamma_s|^2}$$

# A.S., $\Gamma$ , series stub, proof

- Equations for computing the series stub  $\theta_{ss}$ :

$$\cos(\varphi + 2\theta) = |\Gamma_s| \Rightarrow \sin(\varphi + 2\theta) = \pm \sqrt{1 - |\Gamma_s|^2}$$

$$\text{Im}[z_s(\theta)] = \frac{2 \cdot |\Gamma_s| \cdot \sin(\varphi + 2\theta)}{1 - |\Gamma_s|^2} \Rightarrow \text{Im}[z_s(\theta)] = \frac{\pm 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

- two cases:

$$\varphi + 2\theta \in [0^\circ, 180^\circ] \Rightarrow \sin(\varphi + 2\theta) \geq 0$$

$$\begin{cases} \sin(\varphi + 2\theta) = \sqrt{1 - |\Gamma_s|^2} \\ \text{Im}[z_s(\theta)] = \frac{2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} \end{cases}$$

$$\varphi + 2\theta \in (-180^\circ, 0^\circ) \Rightarrow \sin(\varphi + 2\theta) < 0$$
$$\begin{cases} \sin(\varphi + 2\theta) = -\sqrt{1 - |\Gamma_s|^2} \\ \text{Im}[z_s(\theta)] = \frac{-2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} \end{cases}$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

# A.S., $\Gamma$ , series stub, proof

- We prefer (for microstrip) open circuited stub

$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

- The normalized reactance to be introduced to achieve the match
  - $Z(\theta)$  is the impedance seen **towards** the source,  $Z_0$  series with  $j \cdot X$

$$x = \operatorname{Im} \left[ \frac{Z_{in,oc}}{Z_0} \right] = -\cot \beta \cdot l = \operatorname{Im}[z_s(\theta)]$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

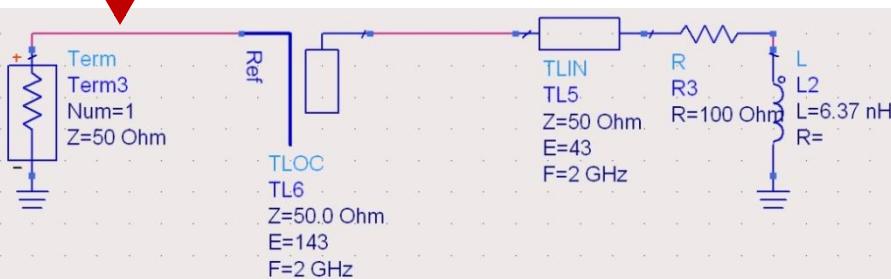
# Analytical solution, $\Gamma$ , usage

$$(\varphi + 2\theta) = \begin{cases} +56.28^\circ \\ -56.28^\circ \end{cases} \quad \theta = \begin{cases} 43.1^\circ \\ 166.8^\circ \end{cases} \quad \text{Im}[z_s(\theta)] = \begin{cases} +1.335 \\ -1.335 \end{cases} \quad \theta_{ss} = \begin{cases} -36.8^\circ + 180^\circ = 143.2^\circ \\ +36.8^\circ \end{cases}$$

- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

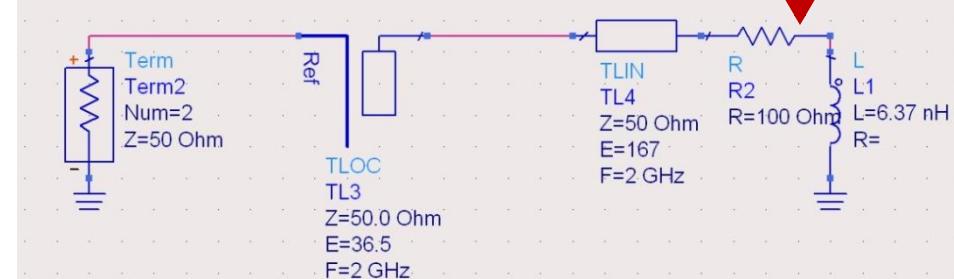
$$l_1 = \frac{43.1^\circ}{360^\circ} \cdot \lambda = 0.120 \cdot \lambda$$

$$l_2 = \frac{143.2^\circ}{360^\circ} \cdot \lambda = 0.398 \cdot \lambda$$



$$l_1 = \frac{166.8^\circ}{360^\circ} \cdot \lambda = 0.463 \cdot \lambda$$

$$l_2 = \frac{36.8^\circ}{360^\circ} \cdot \lambda = 0.102 \cdot \lambda$$

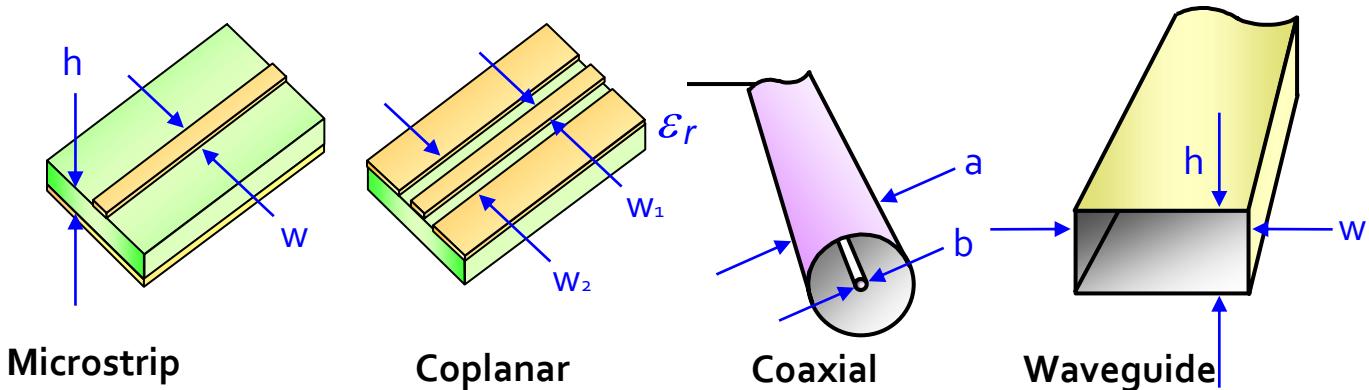


# Single stub tuning

- We choose one of the 8 possible solutions (series/shunt, oc./sc.), taking into account:
  - physical dimensions (area occupied on chip/board)
  - sensitivity of the match on length error ( $\Delta\Gamma/\Delta E$ ,  $\Delta\Gamma/\Delta l$ )
  - convenient frequency behavior (bandwidth)

# Single stub tuning

- We choose one of the 8 possible solutions (series/shunt, oc./sc.), taking into account :
  - physical realizability (in the line technology we use)



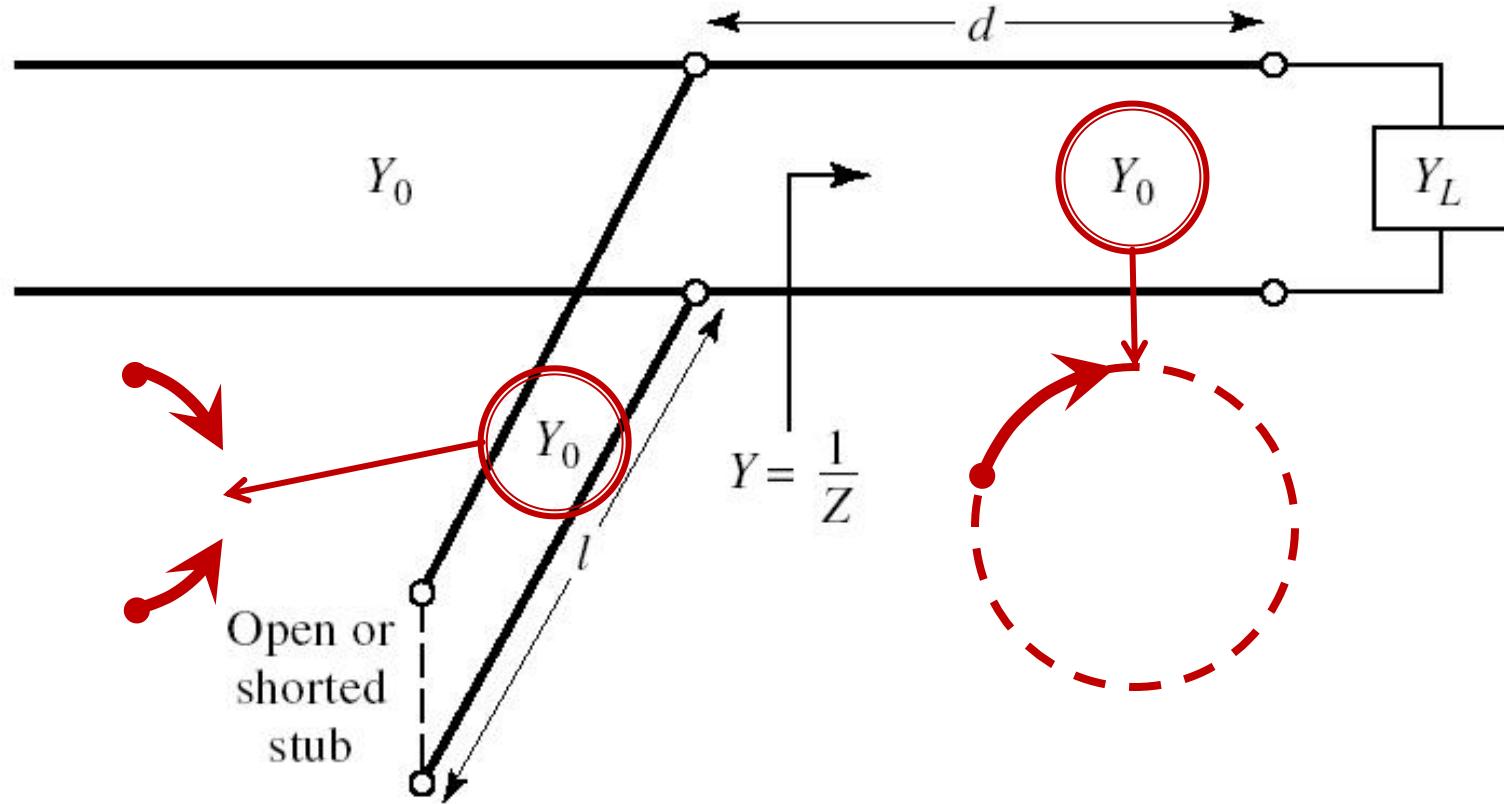
- Main disadvantage:
  - requires a **variable length of line** between the load and the stub

# Analytical solutions

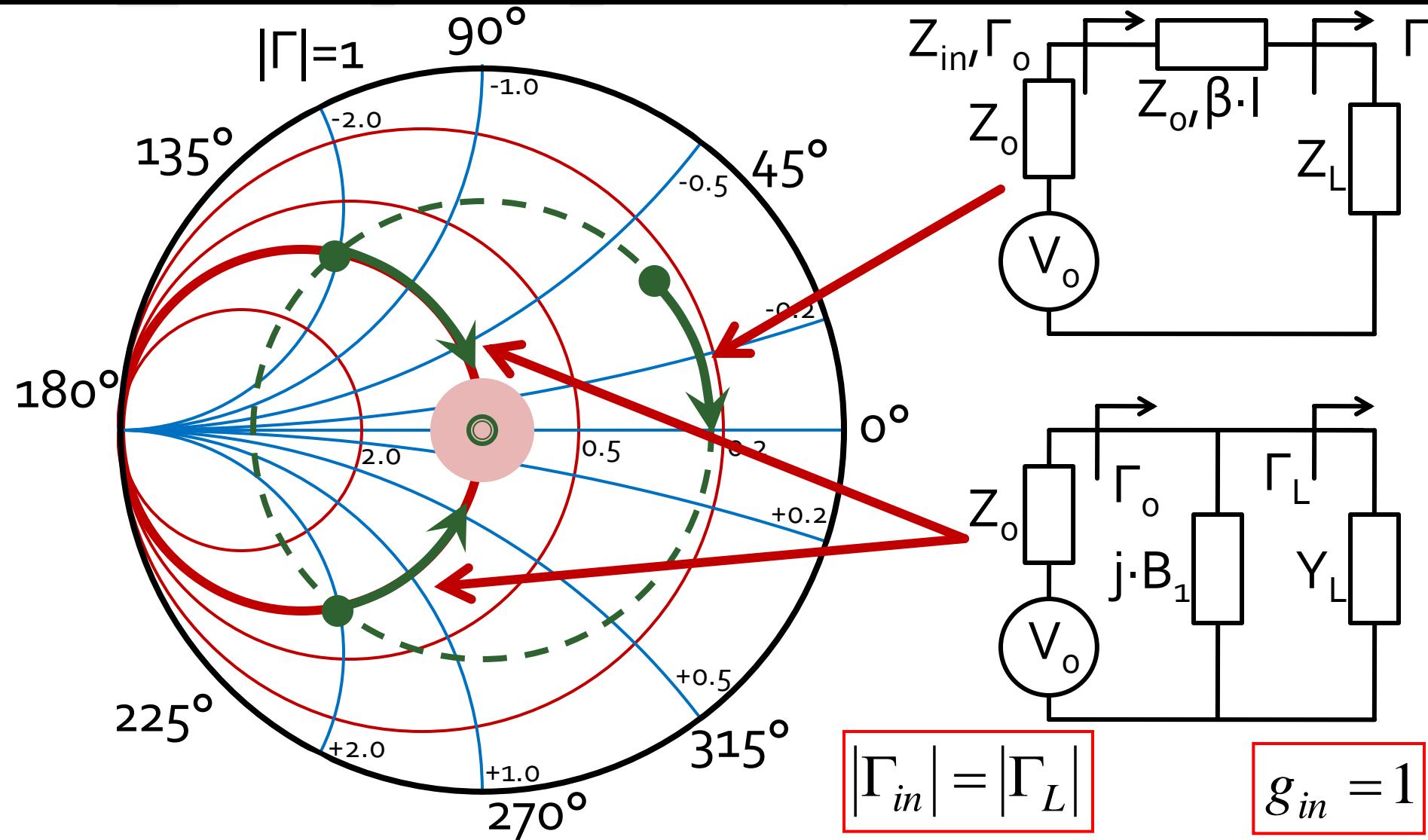
Exam / Project

# Case 1, Shunt Stub

## Shunt Stub



# Matching, series line + shunt susceptance



# Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_s|$$

$$|\Gamma_s| = 0.593 \angle 46.85^\circ$$

$$|\Gamma_s| = 0.593; \quad \varphi = 46.85^\circ \quad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35^\circ$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

- “+” solution** ↓

$$(46.85^\circ + 2\theta) = +126.35^\circ \quad \theta = +39.7^\circ \quad \text{Im } y_s = \frac{-2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = -1.472$$
$$\theta_{sp} = \tan^{-1}(\text{Im } y_s) = -55.8^\circ (+180^\circ) \rightarrow \theta_{sp} = 124.2^\circ$$

- “-” solution** ↓

$$(46.85^\circ + 2\theta) = -126.35^\circ \quad \theta = -86.6^\circ (+180^\circ) \rightarrow \theta = 93.4^\circ$$

$$\text{Im } y_s = \frac{+2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = +1.472 \quad \theta_{sp} = \tan^{-1}(\text{Im } y_s) = 55.8^\circ$$

# Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \text{Im}[y_s(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

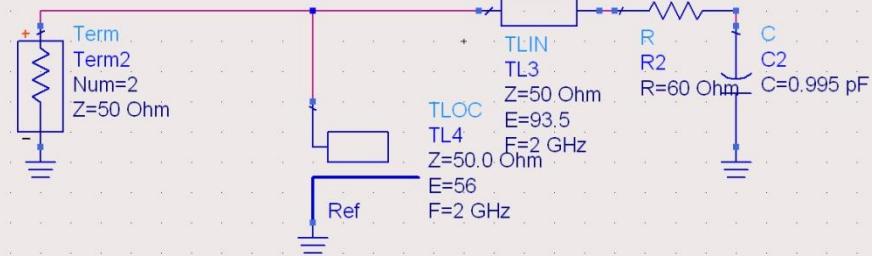
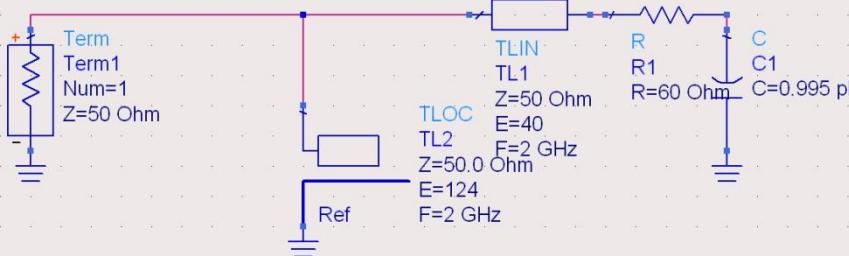
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

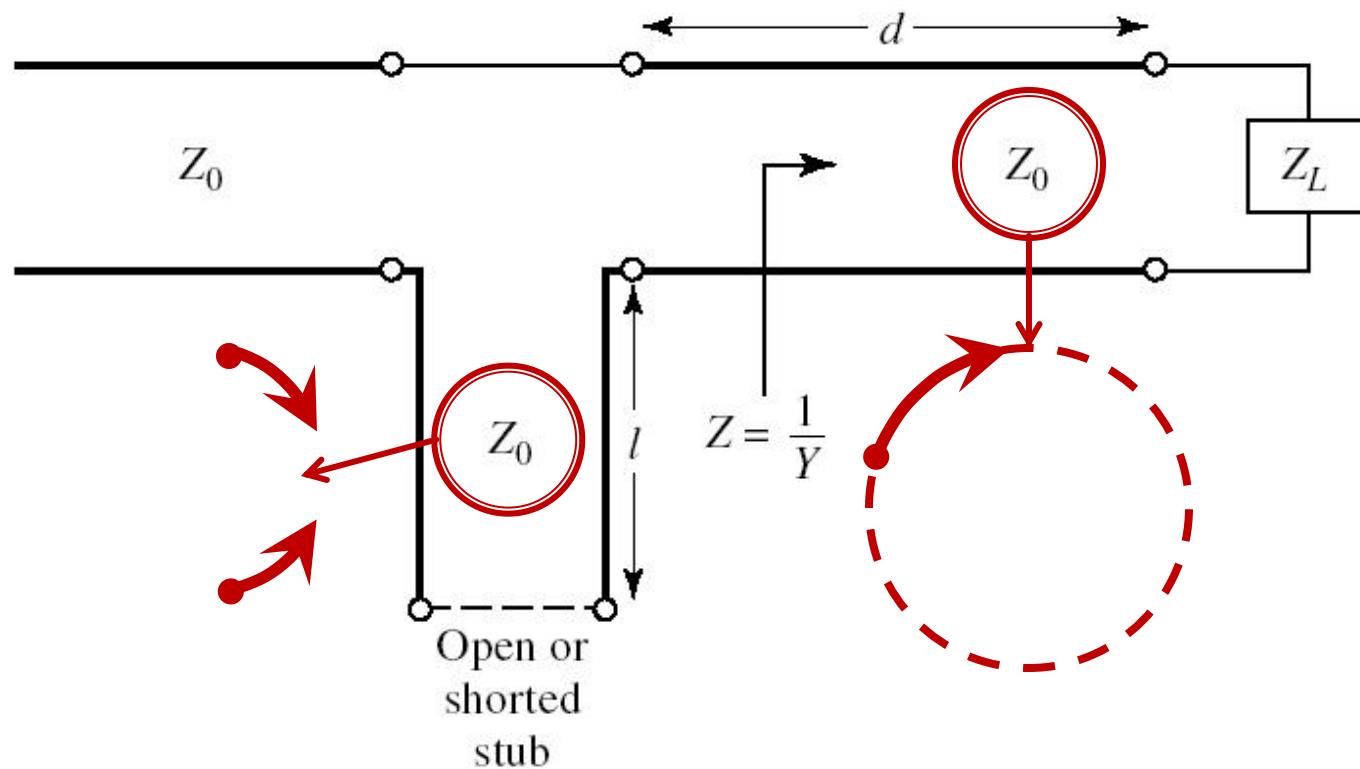
$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$

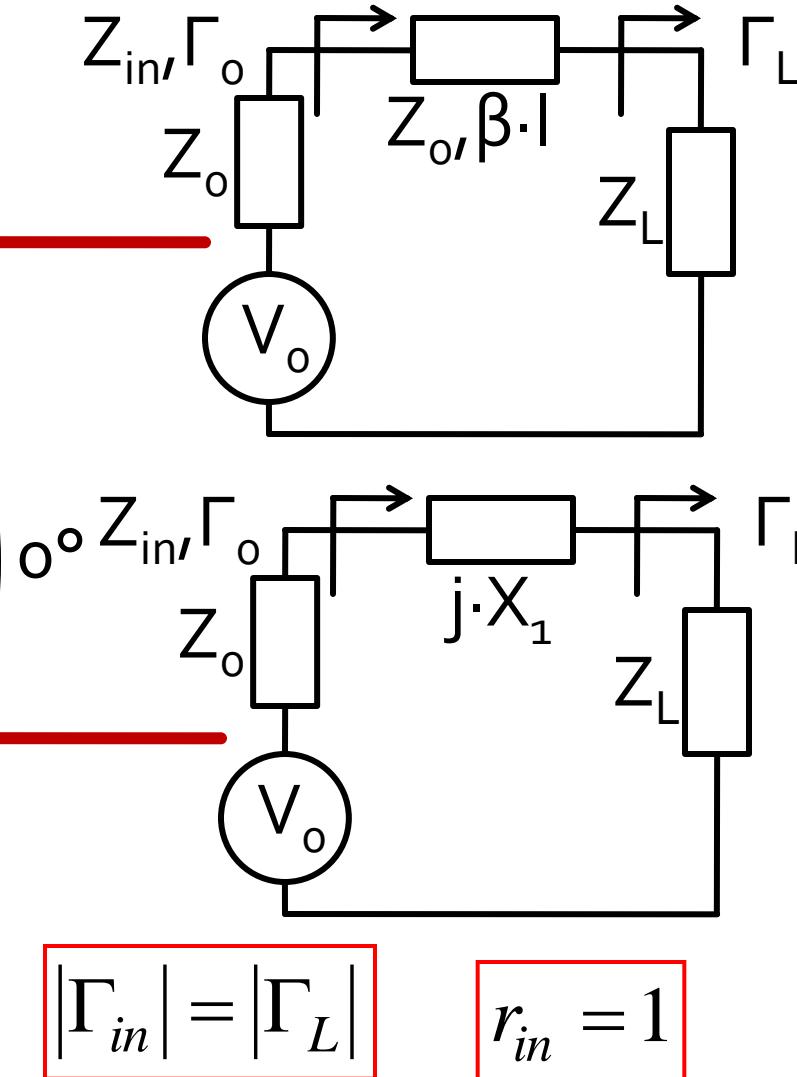
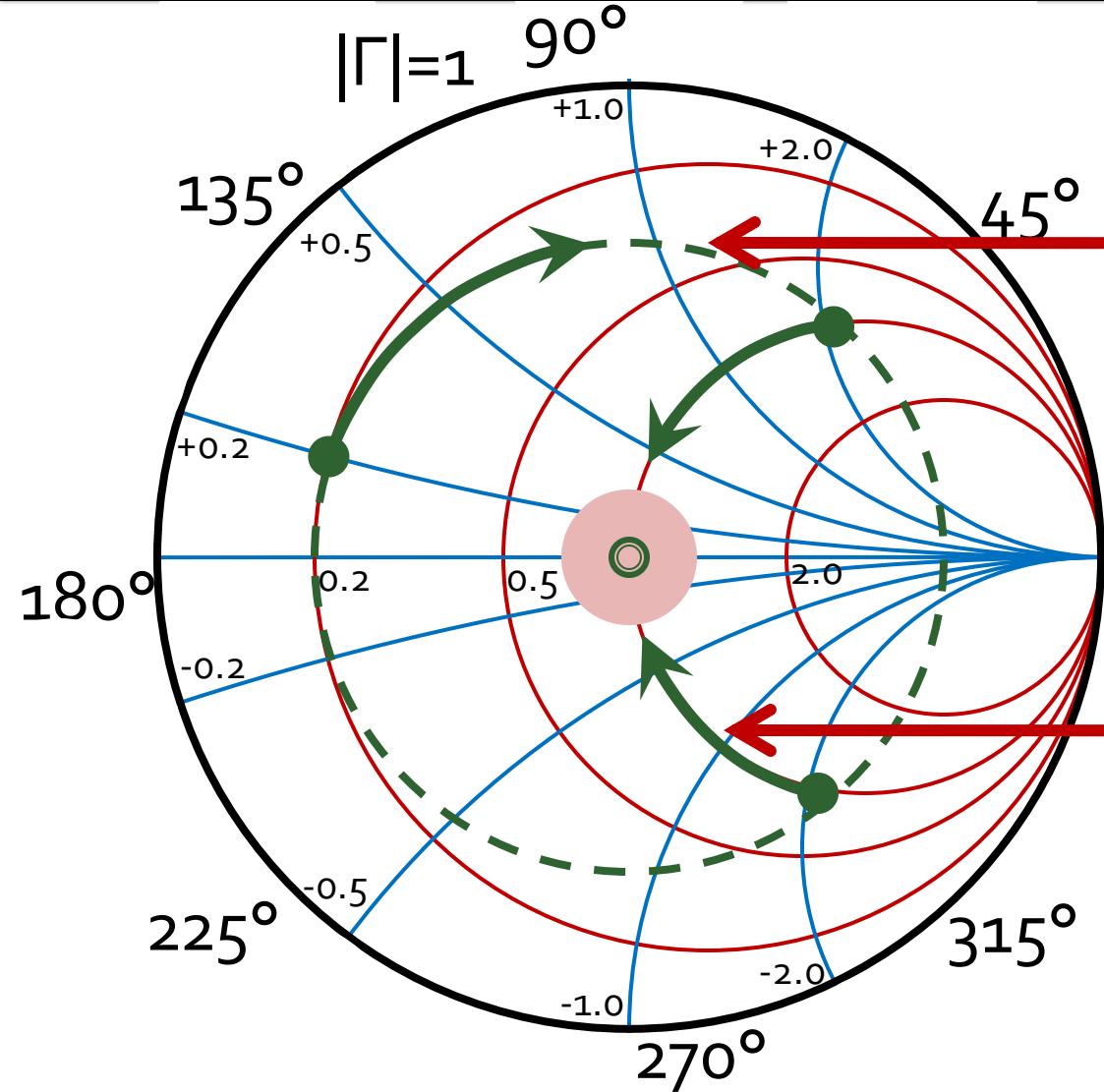


# Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



# Matching, series line + series reactance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$r_{in} = 1$$

# Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_s|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

$$|\Gamma_s| = 0.555 \angle -29.92^\circ$$

$$|\Gamma_s| = 0.555; \quad \varphi = -29.92^\circ \quad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

- “+” solution**

$$(-29.92^\circ + 2\theta) = +56.28^\circ$$

$$\theta = 43.1^\circ$$

$$\text{Im } z_s = \frac{+2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\text{Im } z_s) = -36.8^\circ (+180^\circ) \rightarrow \theta_{ss} = 143.2^\circ$$

- “-” solution**

$$(-29.92^\circ + 2\theta) = -56.28^\circ$$

$$\theta = -13.2^\circ (+180^\circ) \rightarrow \theta = 166.8^\circ$$

$$\text{Im } z_s = \frac{-2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = -1.335$$

$$\theta_{ss} = -\cot^{-1}(\text{Im } z_s) = 36.8^\circ$$

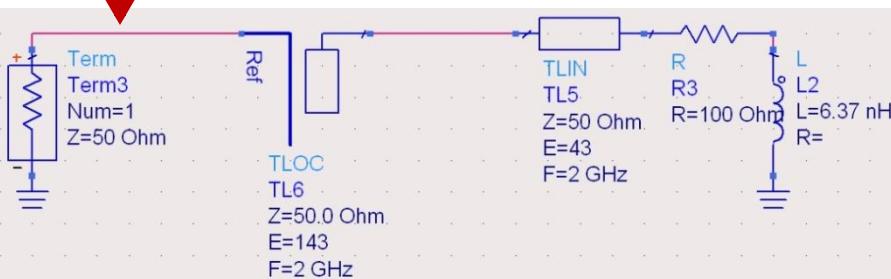
# Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +56.28^\circ \\ -56.28^\circ \end{cases} \quad \theta = \begin{cases} 43.1^\circ \\ 166.8^\circ \end{cases} \quad \text{Im}[z_s(\theta)] = \begin{cases} +1.335 \\ -1.335 \end{cases} \quad \theta_{ss} = \begin{cases} -36.8^\circ + 180^\circ = 143.2^\circ \\ +36.8^\circ \end{cases}$$

- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

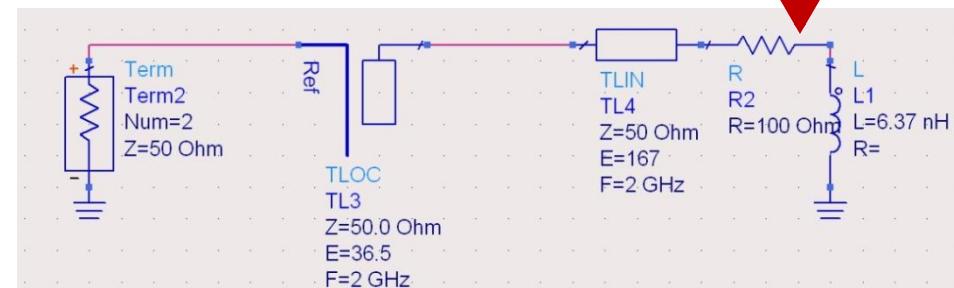
$$l_1 = \frac{43.1^\circ}{360^\circ} \cdot \lambda = 0.120 \cdot \lambda$$

$$l_2 = \frac{143.2^\circ}{360^\circ} \cdot \lambda = 0.398 \cdot \lambda$$



$$l_1 = \frac{166.8^\circ}{360^\circ} \cdot \lambda = 0.463 \cdot \lambda$$

$$l_2 = \frac{36.8^\circ}{360^\circ} \cdot \lambda = 0.102 \cdot \lambda$$



# Stub, observations

- adding or subtracting **180°** ( $\lambda/2$ ) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \quad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

- if the lines/stubs result with **negative** "length"/ "electrical length" we add  $\lambda/2$  /  $180^\circ$  to obtain physically realizable lines
- adding or subtracting **90°** ( $\lambda/4$ ) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \iff Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

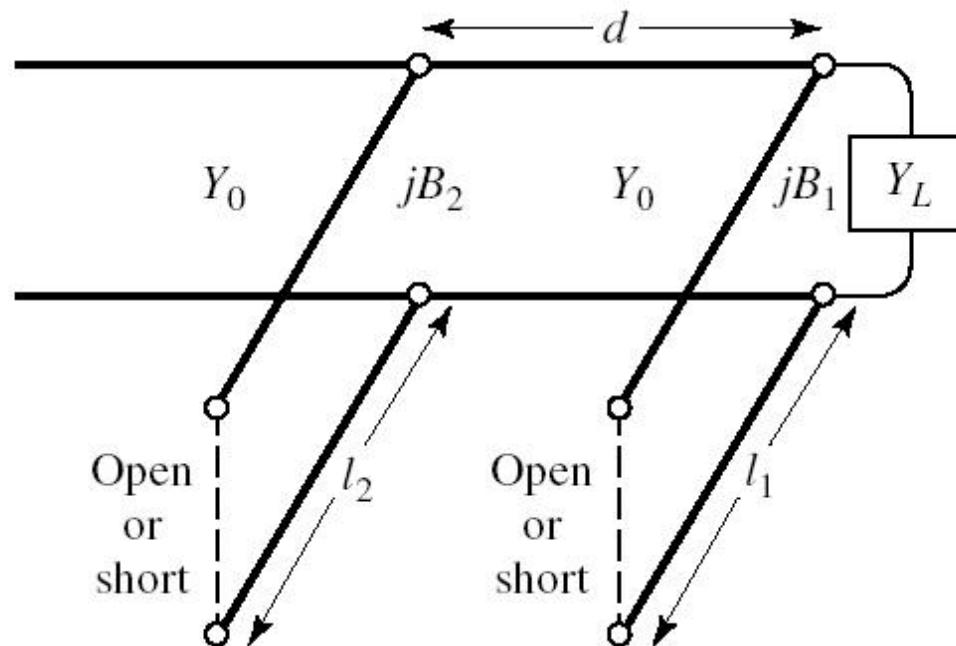
- for the stub we can add or subtract 90° ( $\lambda/4$ ) while in the same time changing **open-circuit**  $\Leftrightarrow$  **short-circuit**

# **Double stub tuning**

Adaptarea cu două secțiuni de linie

# Double stub tuning

- Double stub tuning
- uses two tuning stubs in fixed positions (a fixed length of line between the stubs)



# Double stub tuning

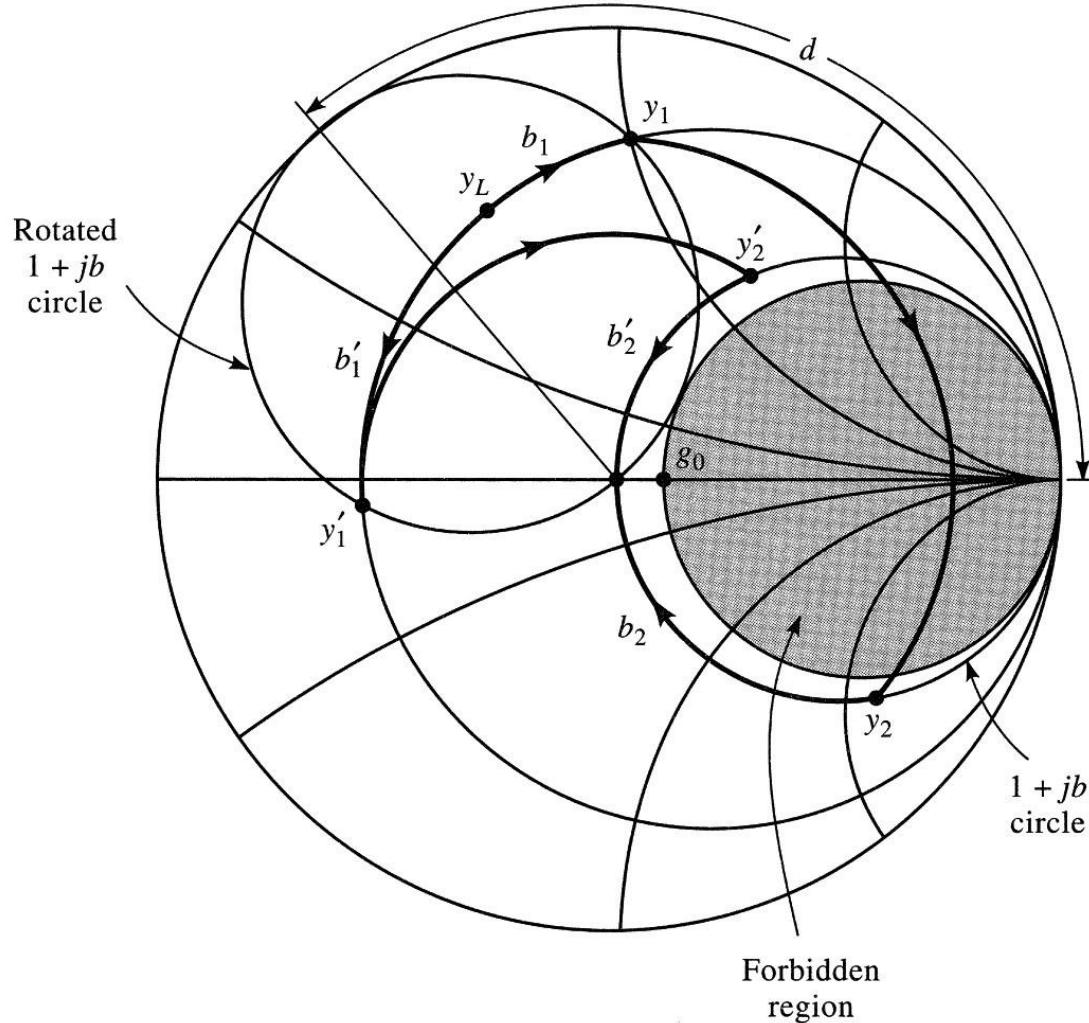
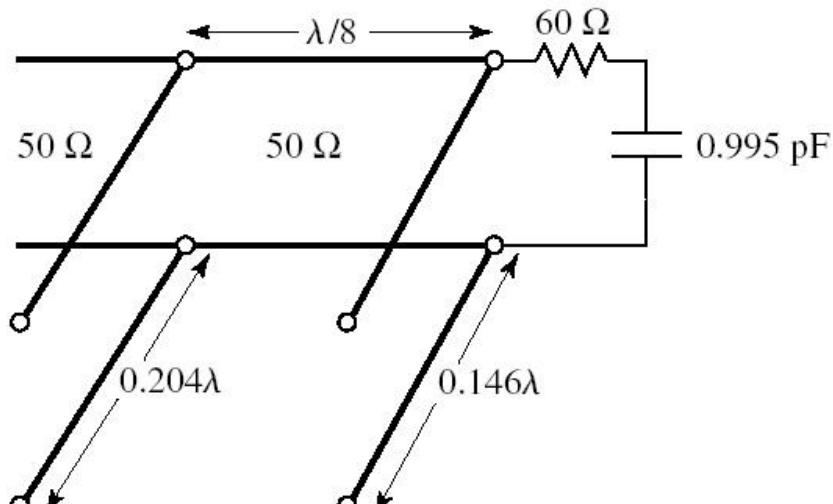


Figure 5.8

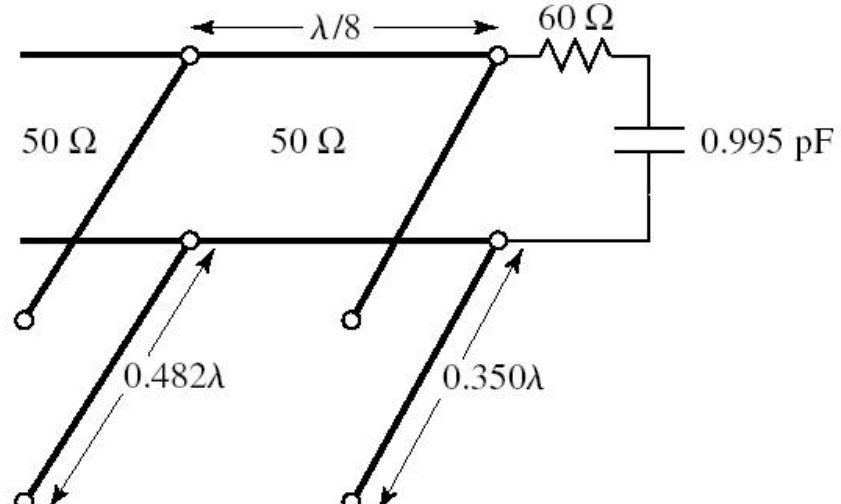
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# Double stub tuning

- same load:  $60 \Omega$  series with  $0.995 \text{ pF}$  at  $2\text{GHz}$
- two possible solutions



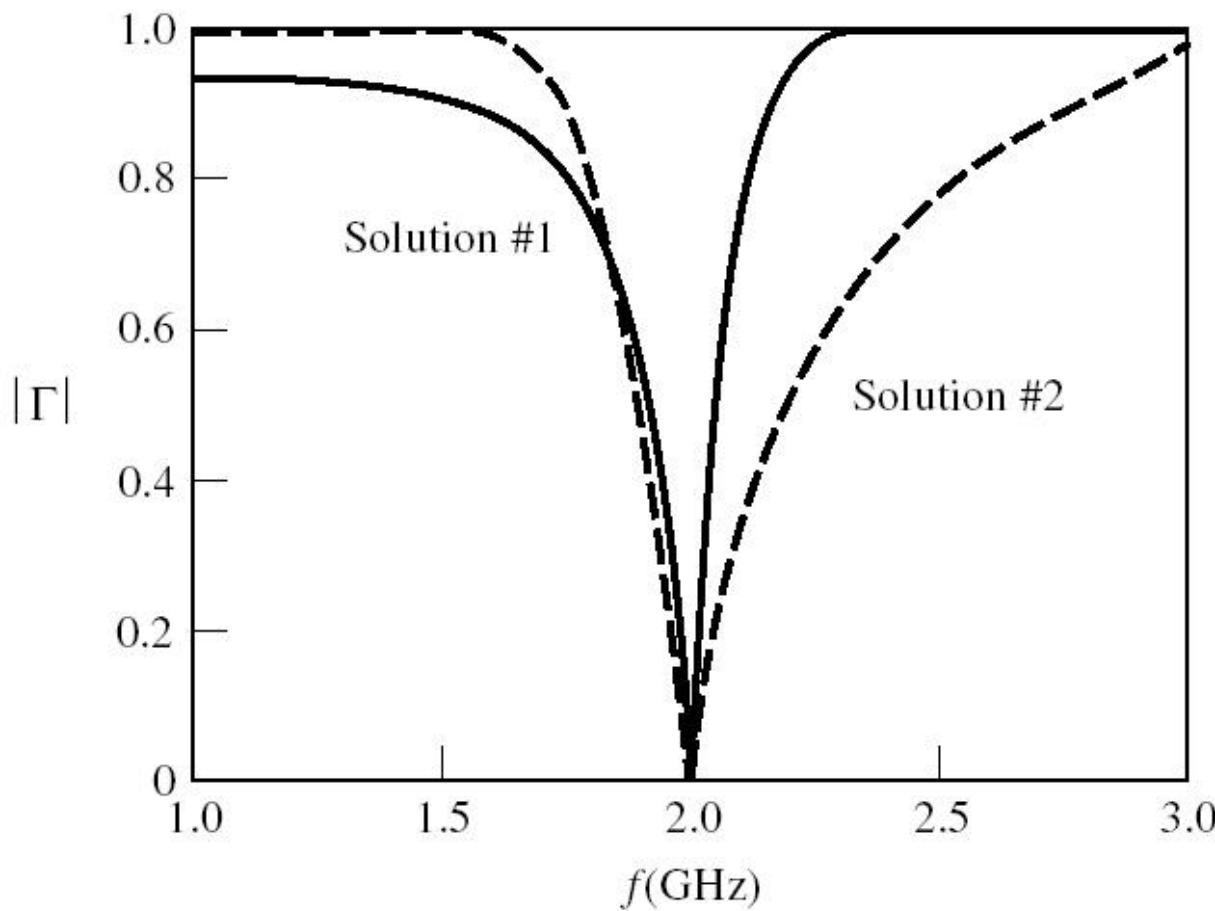
Solution 1



Solution 2

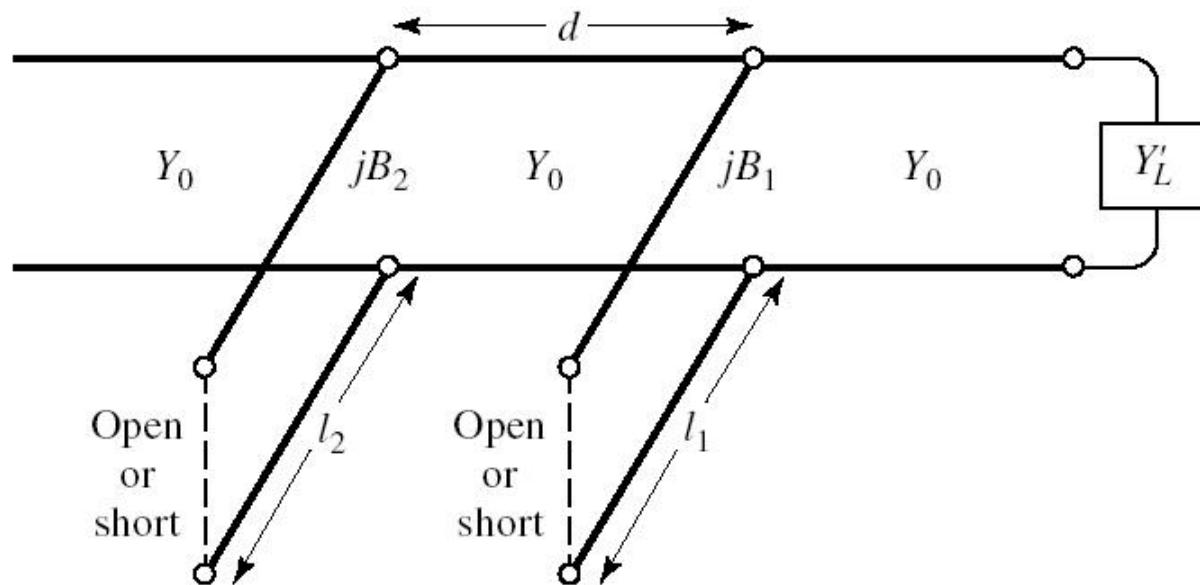
# Double stub tuning

- two possible solutions

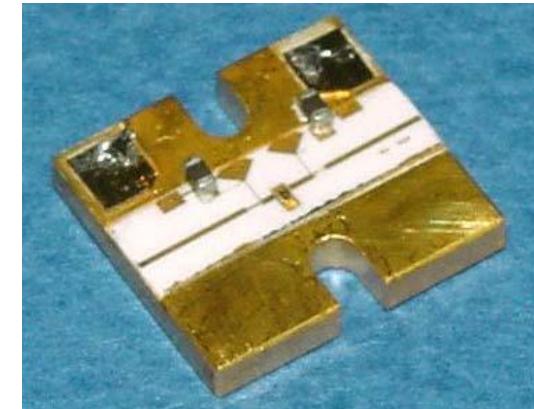
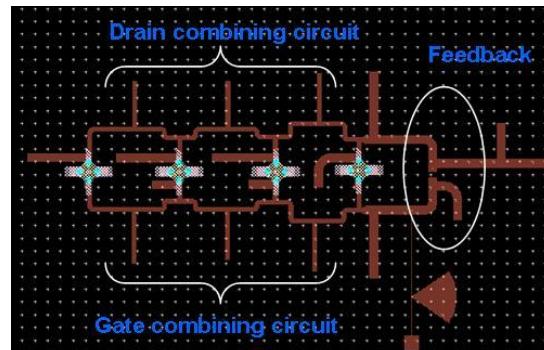
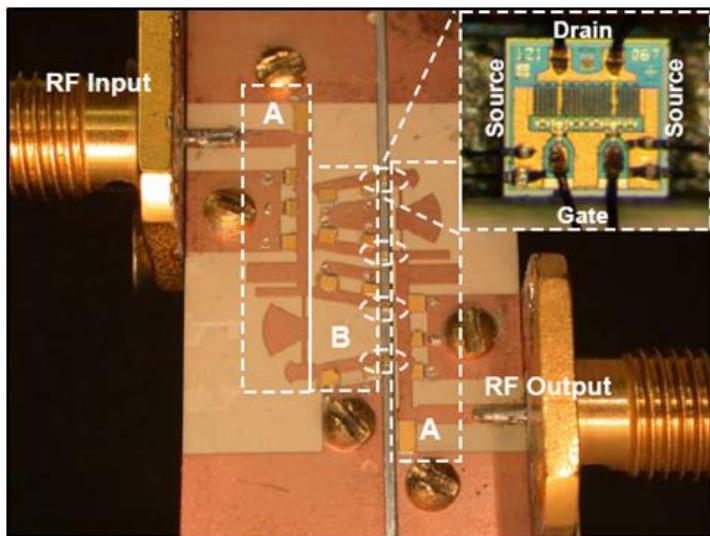
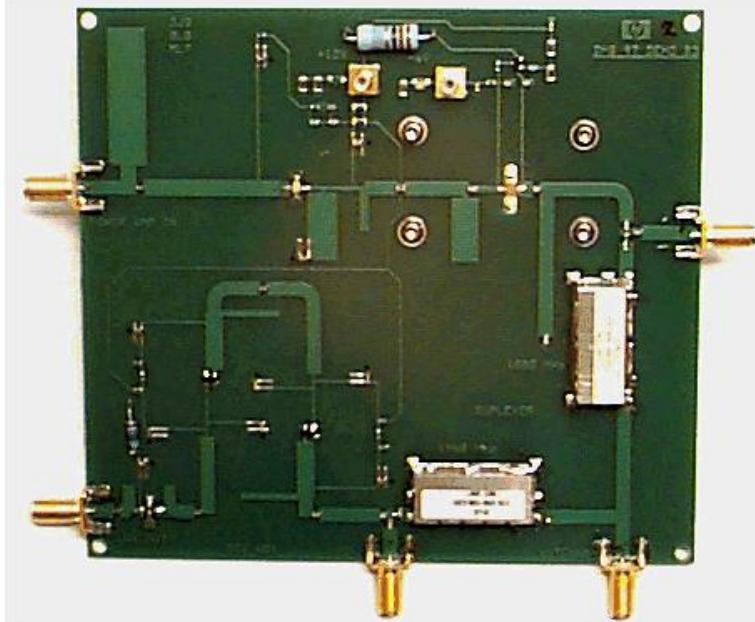
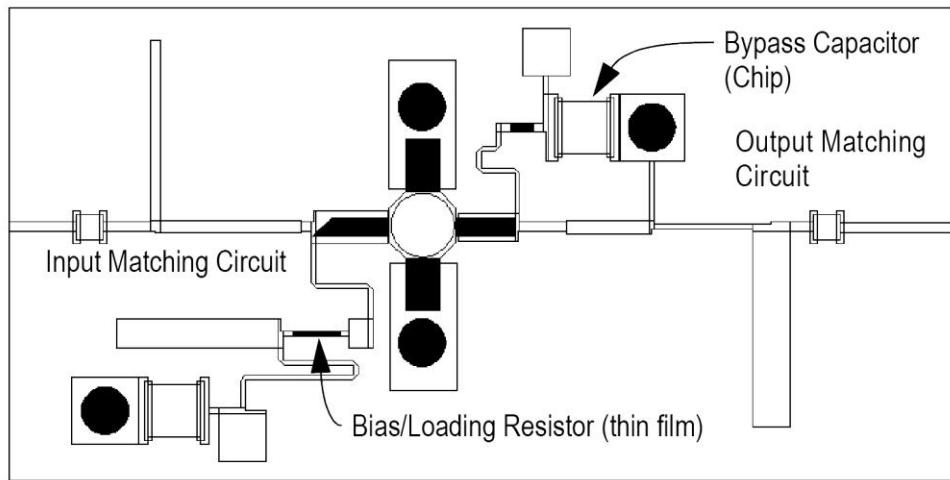


# Double stub tuning

- Typically  $d = \lambda/8$  or  $d = 3\lambda/8$ 
  - $d \rightarrow 0$  and  $d \rightarrow \lambda/2$  offer frequency sensitive solutions
- **Not possible** for every load
  - unless we can add a specific length of line between the load and the first stub

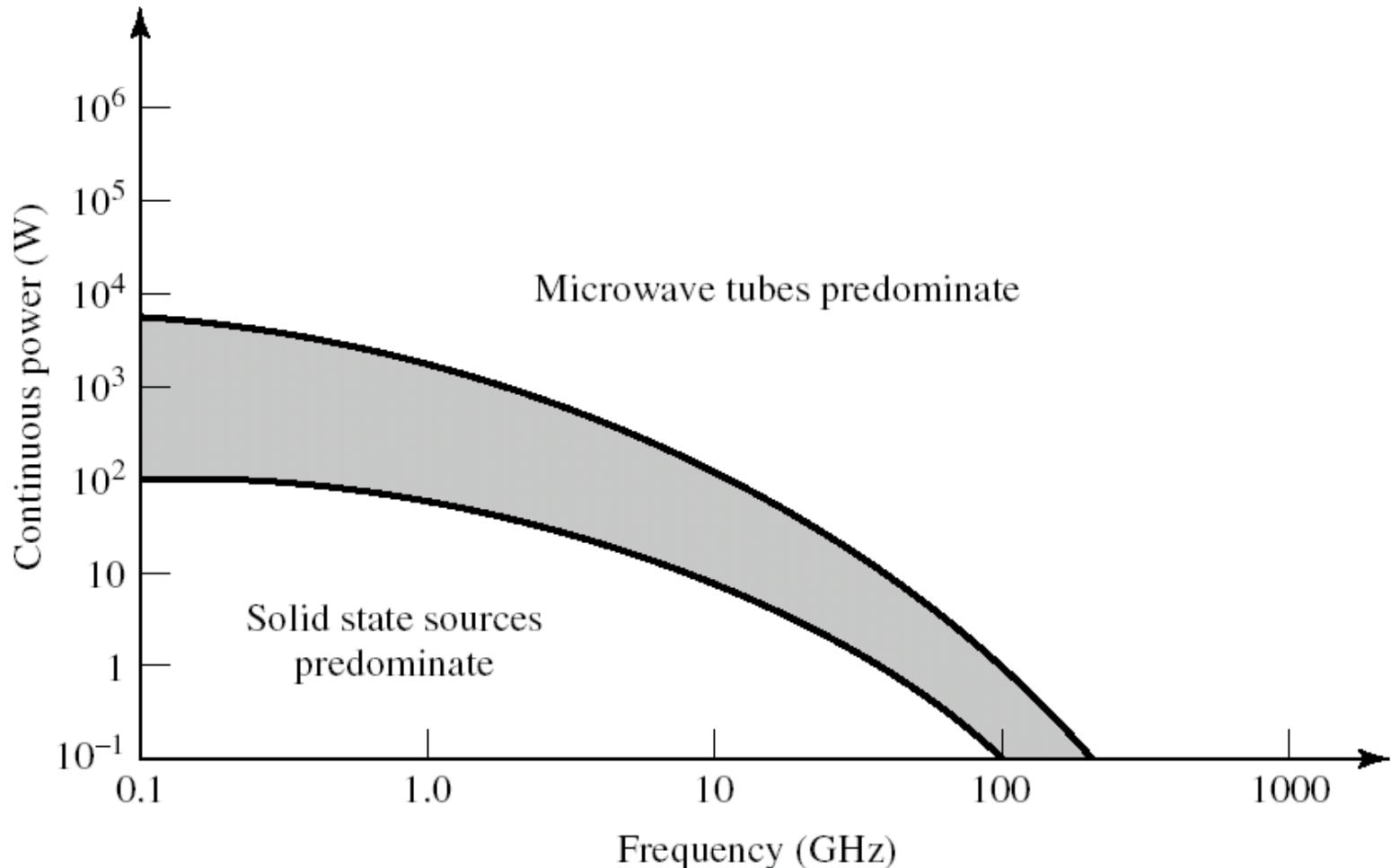


# Impedance Matching with Stubs

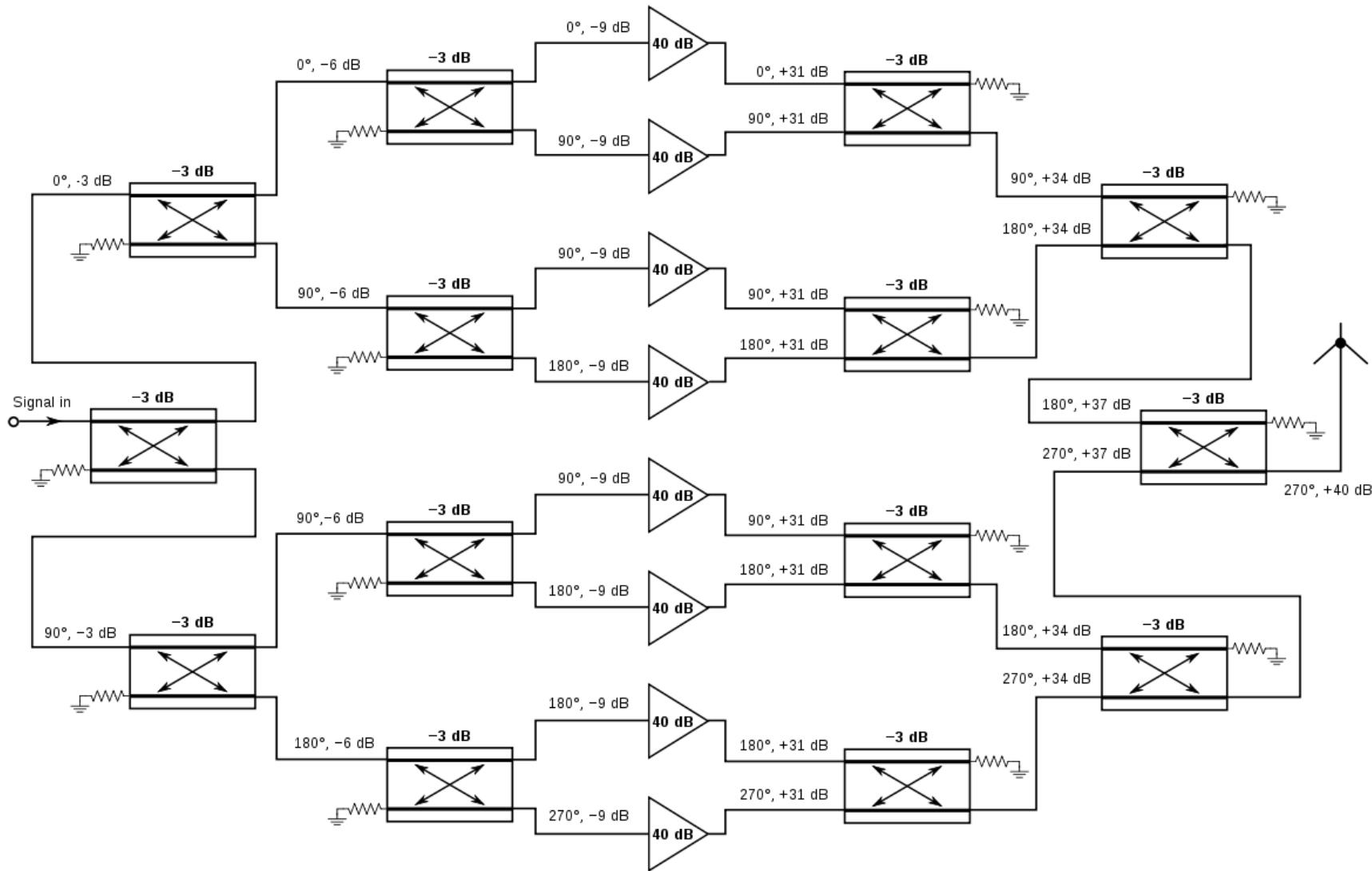


# Microwave Amplifiers

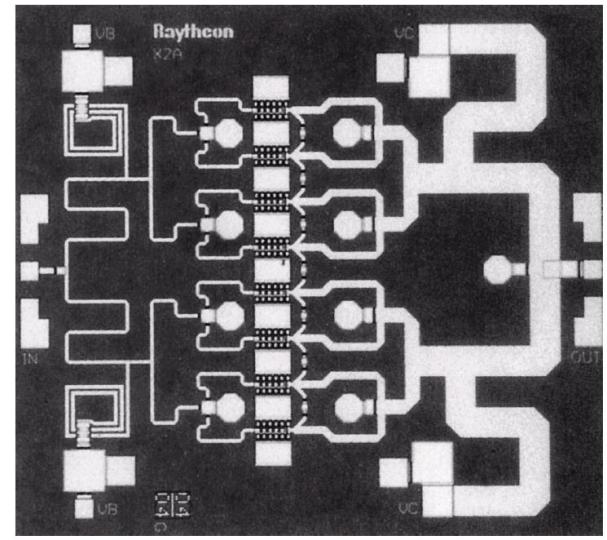
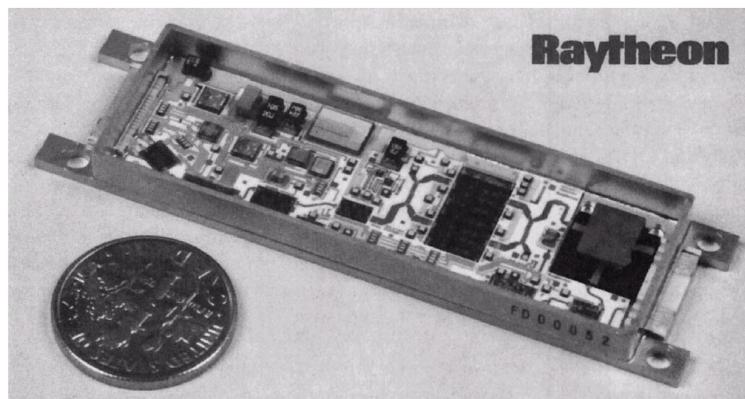
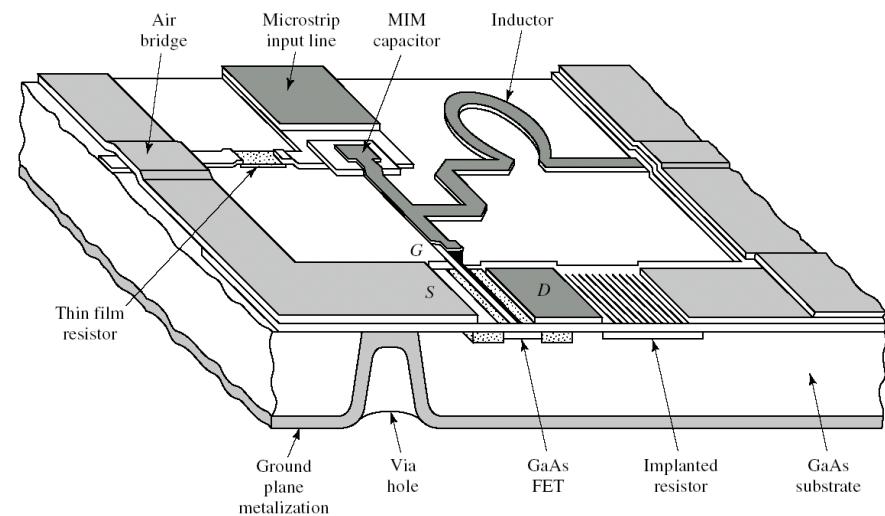
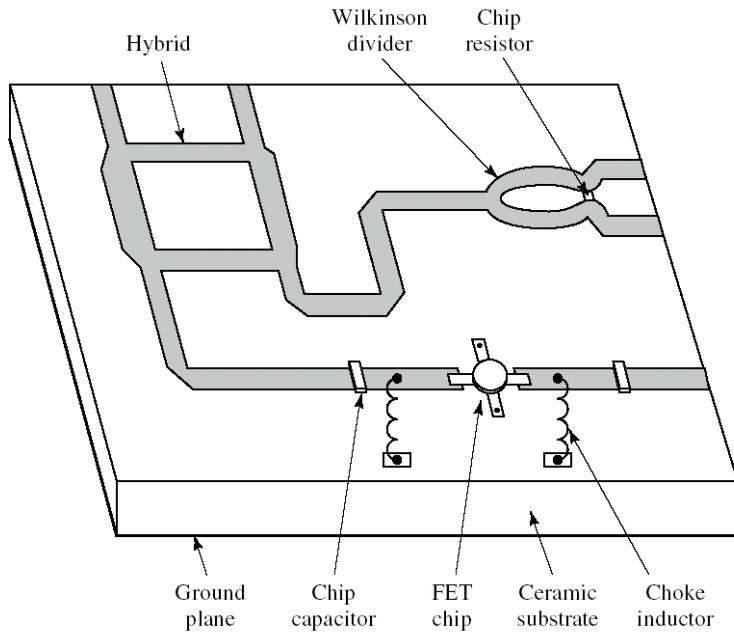
# Microwave Amplifiers



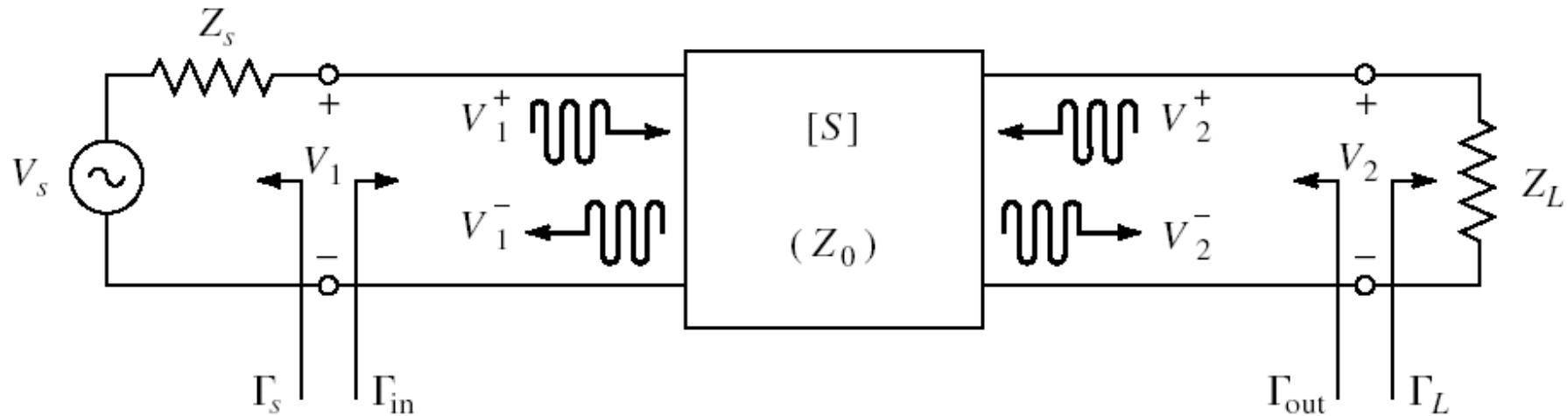
# Balanced amplifiers



# Microwave Integrated Circuits



# Amplifier as two-port



- Charaterized with S parameters
- normalized at  $Z_0$  (implicit  $50\Omega$ )
- Datasheets: S parameters for specific bias conditions

# Datasheets

CEL

## NE46100 / NE46134

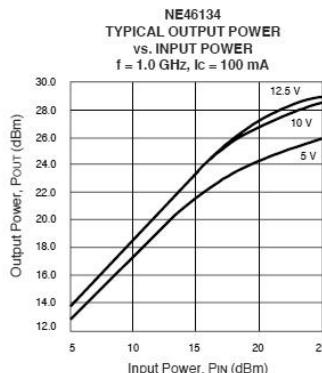
### NPN MEDIUM POWER MICROWAVE TRANSISTOR

#### FEATURES

- HIGH DYNAMIC RANGE
- LOW IM DISTORTION: -40 dBc
- HIGH OUTPUT POWER : 27.5 dBm at TYP
- LOW NOISE: 1.5 dB TYP at 500 MHz
- LOW COST

#### DESCRIPTION

The NE461 series of NPN silicon epitaxial bipolar transistors is designed for medium power applications requiring high dynamic range. This device exhibits an outstanding combination of high gain and low intermodulation distortion, as well as low noise figure. The NE461 series offers excellent performance and reliability at low cost through titanium, platinum, gold metallization system and direct nitride passivation of the surface of the chip. Devices are available in a low cost surface mount package (SOT-89) as well as in chip form.



#### ELECTRICAL CHARACTERISTICS ( $T_A = 25^\circ\text{C}$ )

PART NUMBER EIAJ REGISTERED NUMBER PACKAGE OUTLINE			NE46100 00 (CHIP)			NE46134 2SC4536 34		
SYMBOLS	PARAMETERS AND CONDITIONS	UNITS	MIN	TYP	MAX	MIN	TYP	MAX
$f_T$	Gain Bandwidth Product at $V_{CE} = 10 \text{ V}$ , $I_C = 100 \text{ mA}$	GHz		5.5		5.5		
$NF_{MIN}$	Minimum Noise Figure <sup>3</sup> at $V_{CE} = 10 \text{ V}$ , $I_C = 50 \text{ mA}$ , 500 MHz $V_{CE} = 10 \text{ V}$ , $I_C = 50 \text{ mA}$ , 1 GHz	dB		1.5		1.5		
$G_L$	Linear Gain, $V_{CE} = 12.5 \text{ V}$ , $I_C = 100 \text{ mA}$ , 2.0 GHz $V_{CE} = 12.5 \text{ V}$ , $I_C = 100 \text{ mA}$ , 1.0 GHz	dB		9.0		8.0		
$IS_{21EI}^2$	Insertion Power Gain at 10 V, 50 mA, $f = 1.0 \text{ GHz}$	dB		10.0		5.5	7.0	
$h_{FE}$	DC Current Gain <sup>2</sup> at $V_{CE} = 10 \text{ V}$ , $I_C = 50 \text{ mA}$		40	200	40		200	
$I_{CBO}$	Collector Cutoff Current at $V_{CB} = 20 \text{ V}$ , $I_E = 0 \text{ mA}$	mA		5.0		5.0		
$I_{EB0}$	Emitter Cutoff Current at $V_{EB} = 2 \text{ V}$ , $I_C = 0 \text{ mA}$	mA		5.0		5.0		
$P_{1dB}$	Output Power at 1 dB Compression, $V_{CE} = 12.5 \text{ V}$ , $I_C = 100 \text{ mA}$ , 2.0 GHz $V_{CE} = 12.5 \text{ V}$ , $I_C = 100 \text{ mA}$ , 1.0 GHz	dBm	27.0			27.5		
$IM_3$	Intermodulation Distortion, 10 V, 100 mA, $F_1 = 1.0 \text{ GHz}$ , $F_2 = 0.99 \text{ GHz}$							

# Datasheets

**NE46100**

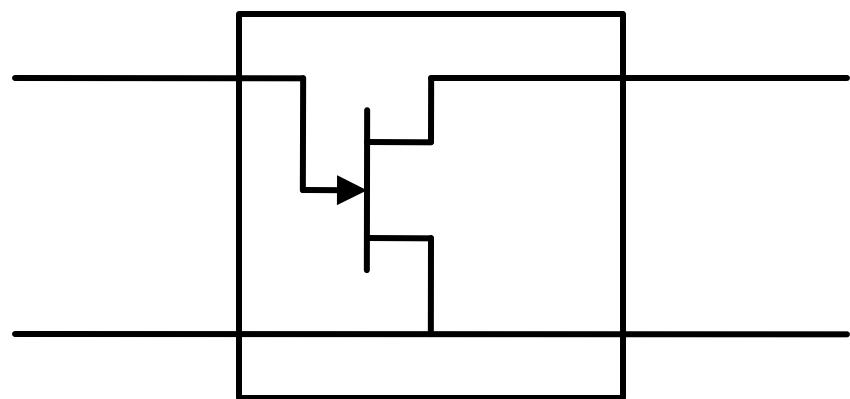
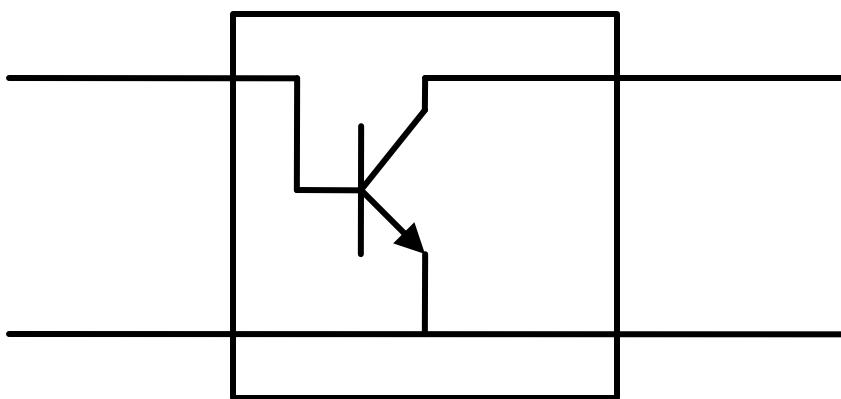
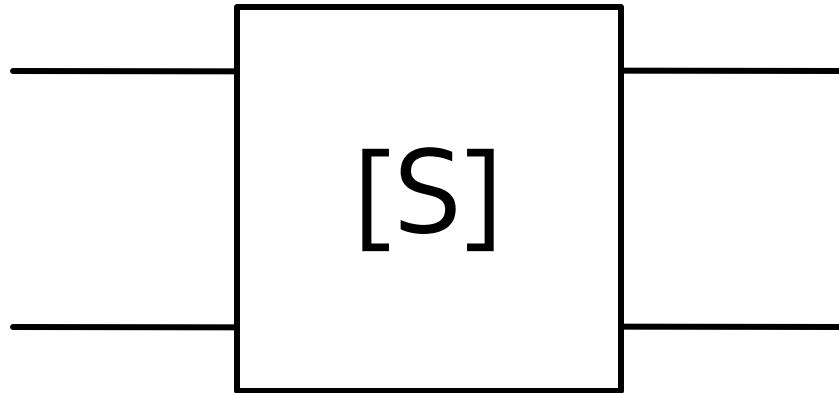
**VCE = 5 V, Ic = 50 mA**

FREQUENCY (MHz)	S <sub>11</sub>		S <sub>21</sub>		S <sub>12</sub>		S <sub>22</sub>		K	MAG <sup>2</sup> (dB)
	MAG	ANG	MAG	ANG	MAG	ANG	MAG	ANG		
100	0.778	-137	26.776	114	0.028	30	0.555	-102	0.16	29.8
200	0.815	-159	14.407	100	0.035	29	0.434	-135	0.36	26.2
500	0.826	-177	5.855	84	0.040	38	0.400	-162	0.75	21.7
800	0.827	176	3.682	76	0.052	43	0.402	-169	0.91	18.5
1000	0.826	173	2.963	71	0.058	47	0.405	-172	1.02	16.3
1200	0.825	170	2.441	66	0.064	47	0.412	-174	1.08	14.0
1400	0.820	167	2.111	61	0.069	47	0.413	-176	1.17	12.4
1600	0.828	165	1.863	57	0.078	54	0.426	-177	1.15	11.4
1800	0.827	162	1.671	53	0.087	50	0.432	-178	1.14	10.6
2000	0.828	159	1.484	49	0.093	50	0.431	-180	1.17	9.5
2500	0.822	153	1.218	39	0.11	48	0.462	177	1.18	7.8
3000	0.818	148	1.010	30	0.135	46	0.490	174	1.16	6.3
3500	0.824	142	0.876	21	0.147	44	0.507	170	1.16	5.3
4000	0.812	137	0.762	13	0.168	38	0.535	167	1.14	4.3

**VCE = 5 V, Ic = 100 mA**

100	0.778	-144	27.669	111	0.027	35	0.523	-114	0.27	30.2
200	0.820	-164	14.559	97	0.029	29	0.445	-144	0.42	27.0
500	0.832	-179	5.885	84	0.035	38	0.435	-166	0.81	22.2
800	0.833	175	3.691	76	0.048	45	0.435	-173	0.95	18.8
1000	0.831	172	2.980	71	0.056	51	0.437	-176	1.05	16.0
1200	0.836	169	2.464	67	0.061	52	0.432	-178	1.11	14.0
1400	0.829	166	2.121	61	0.072	53	0.447	-180	1.12	12.6
1600	0.831	164	1.867	58	0.080	54	0.445	179	1.14	11.4

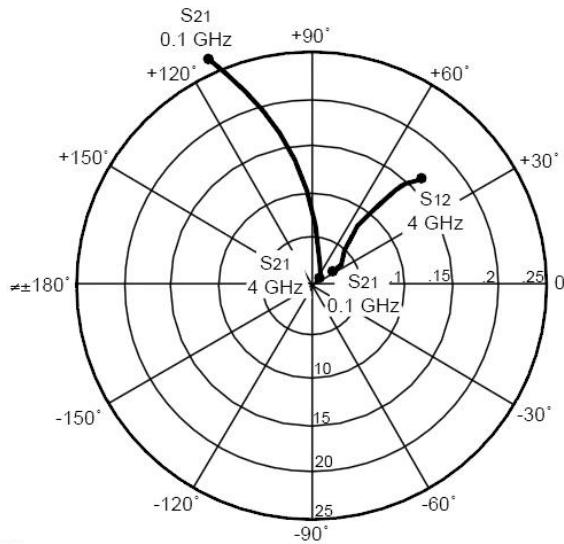
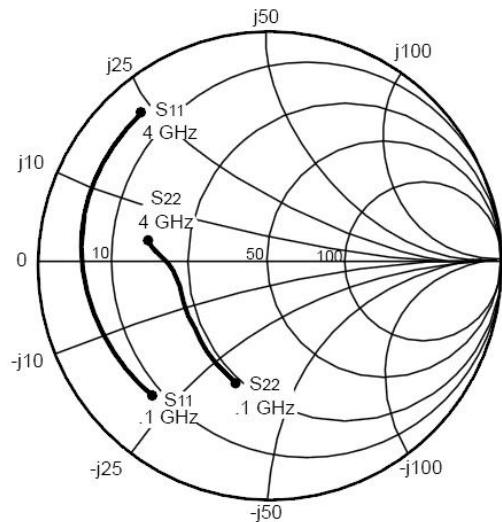
# S parameters for transistors



# Datasheets

NE46100, NE46134

## TYPICAL COMMON EMITTER SCATTERING PARAMETERS<sup>1</sup> ( $T_A = 25^\circ\text{C}$ )



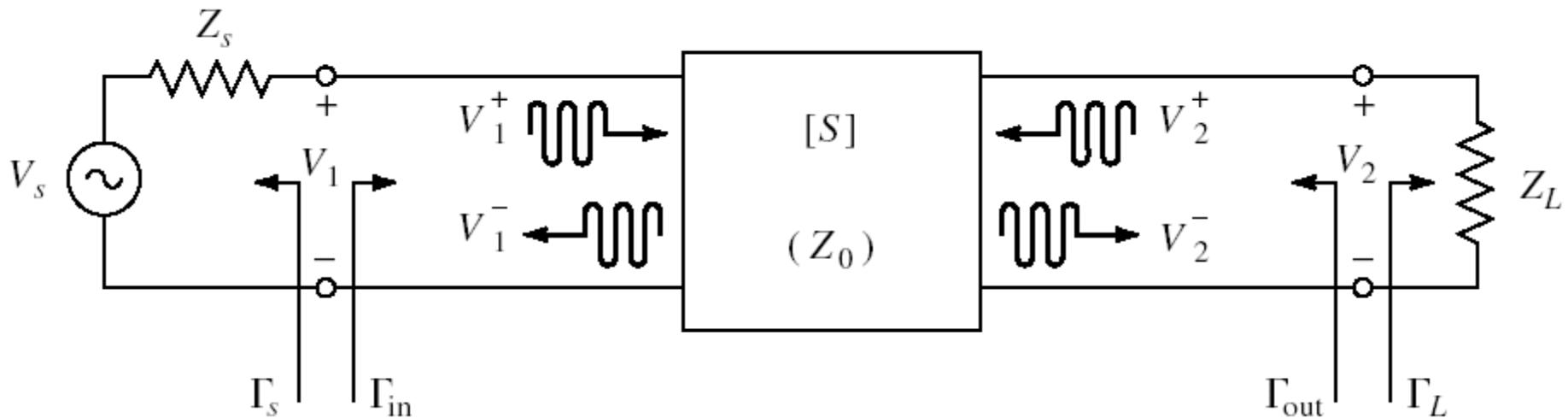
Coordinates in Ohms  
Frequency in GHz  
 $V_{CE} = 5 \text{ V}, I_C = 50 \text{ mA}$

# S<sub>2</sub>P - Touchstone

- Touchstone file format (\*.s2p)

```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V  ID = 15 mA
# GHz S MA R 50
! f   S11      S21      S12      S22
! GHz  MAG  ANG  MAG  ANG  MAG  ANG  MAG  ANG
1.000 0.9800 -18.0  2.230 157.0  0.0240  74.0  0.6900 -15.0
2.000 0.9500 -39.0  2.220 136.0  0.0450  57.0  0.6600 -30.0
3.000 0.8900 -64.0  2.210 110.0  0.0680  40.0  0.6100 -45.0
4.000 0.8200 -89.0  2.230  86.0  0.0850  23.0  0.5600 -62.0
5.000 0.7400 -115.0 2.190  61.0  0.0990  7.0   0.4900 -80.0
6.000 0.6500 -142.0 2.110  36.0  0.1070 -10.0  0.4100 -98.0
!
! f   Fmin  Gammaopt rn/50
! GHz  dB  MAG  ANG  -
2.000  1.00 0.72 27  0.84
4.000  1.40 0.64 61  0.58
```

# Amplifier as two-port



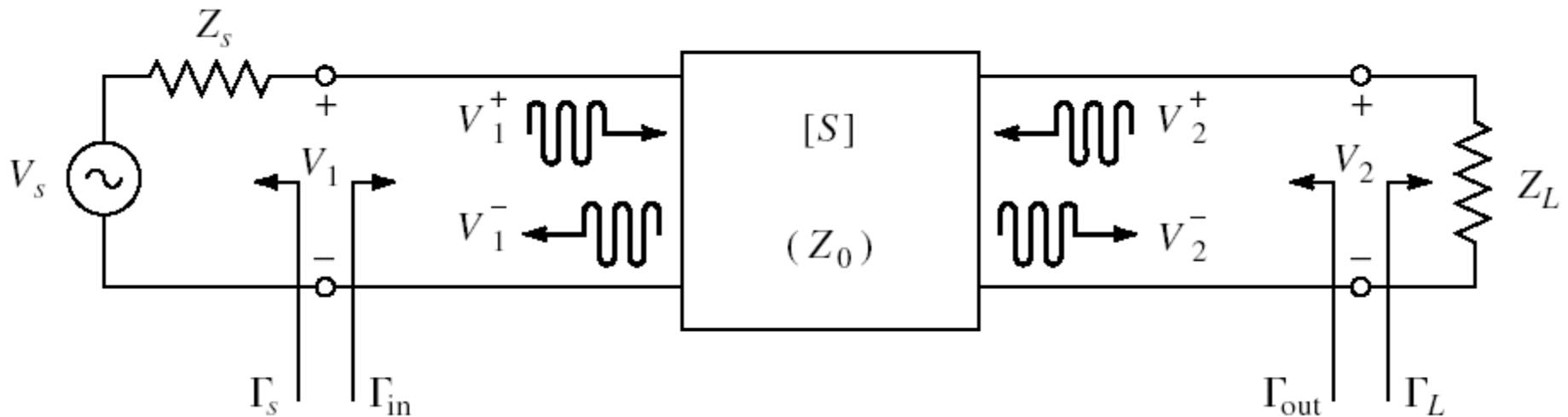
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \quad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$\Gamma_L = \frac{V_2^+}{V_2^-}$$

$$V_1^- = S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ = S_{11} \cdot V_1^+ + S_{12} \cdot \Gamma_L \cdot V_2^-$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

# Amplifier as two-port



$$V_1^- = S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ = S_{11} \cdot V_1^+ + S_{12} \cdot \Gamma_L \cdot V_2^-$$

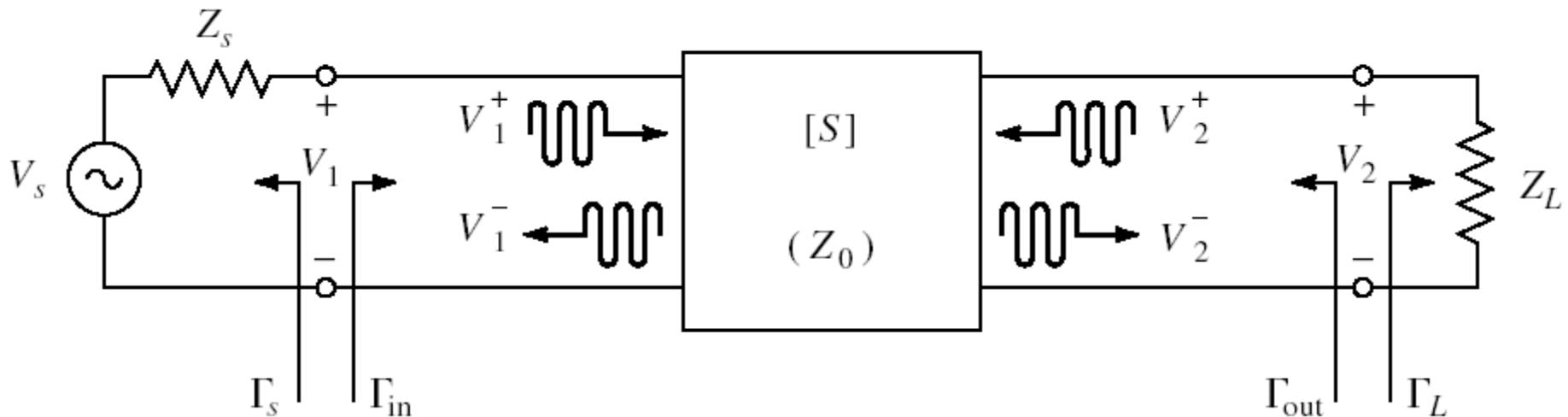
$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

■ similarly

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

# Amplifier as two-port

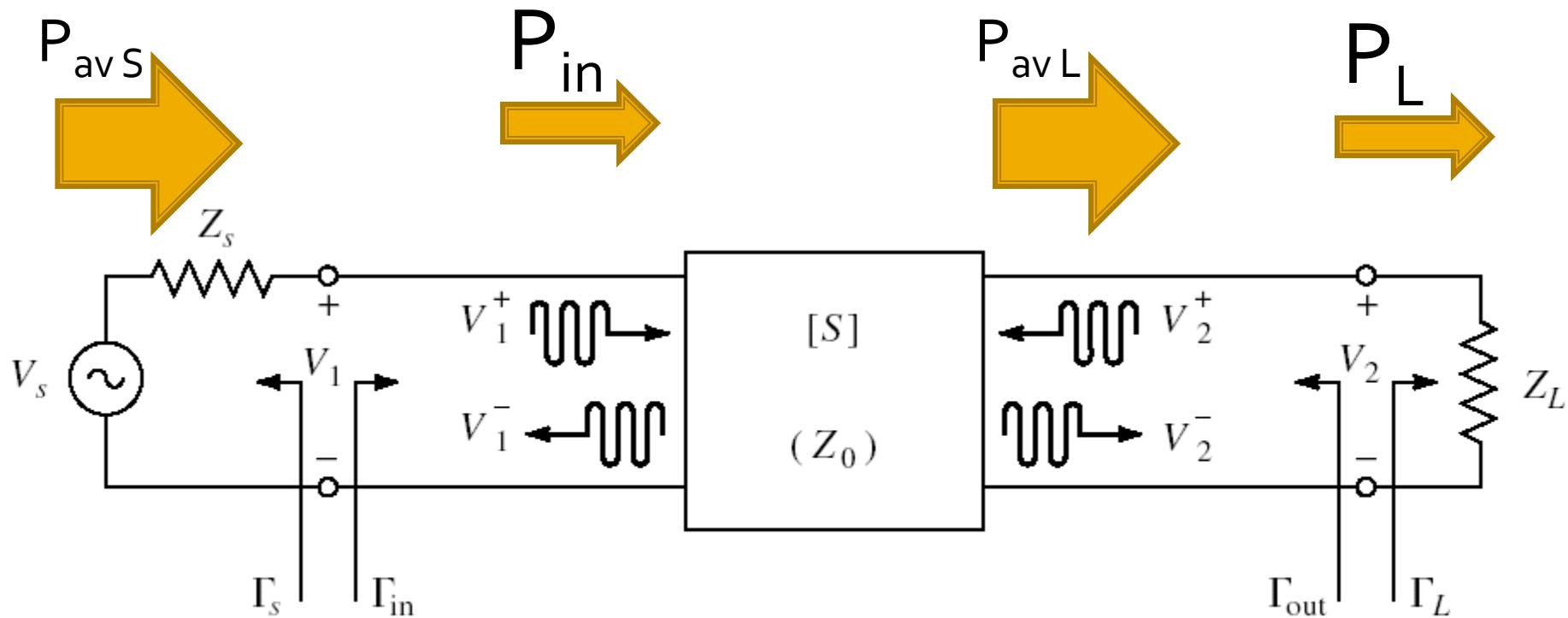


$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

# Power / Matching

- Two ports in which matching influences the power transfer



# Signal power

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$V_1 = \frac{V_S \cdot Z_{in}}{Z_S + Z_{in}} = V_1^+ + V_1^- = V_1^+ \cdot (1 + \Gamma_{in})$$

■ L2       $P_{in} = \frac{1}{2 \cdot Z_0} \cdot |V_1^+|^2 \cdot (1 - |\Gamma_{in}|^2)$

$$P_{in} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2} (1 - |\Gamma_{in}|^2)$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

$$P_L = \frac{|V_1^+|^2}{2 \cdot Z_0} \cdot \frac{|S_{21}|^2}{|1 - S_{22} \cdot \Gamma_L|^2} (1 - |\Gamma_L|^2)$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$V_1^+ = \frac{V_S}{2} \frac{(1 - \Gamma_S)}{(1 - \Gamma_S \cdot \Gamma_{in})}$$

$$P_L = \frac{1}{2 \cdot Z_0} \cdot |V_2^-|^2 \cdot (1 - |\Gamma_L|^2)$$

$$V_2^- = \frac{S_{21} \cdot V_1^+}{1 - S_{22} \cdot \Gamma_L}$$

$$P_L = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{|1 - S_{22} \cdot \Gamma_L|^2} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2}$$

# Signal power

- Signal power

$$P_{in} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2} \left(1 - |\Gamma_{in}|^2\right)$$

$$P_L = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{|1 - S_{22} \cdot \Gamma_L|^2} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2}$$

- Power available from the source

$$P_{av\ S} = P_{in} \Big|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{\left(1 - |\Gamma_S|^2\right)}$$

- Power available on the load (from the network)

$$P_{av\ L} = P_L \Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot |1 - \Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2 \cdot \left(1 - |\Gamma_{out}|^2\right)}$$

# Two-Port Power Gains

## ■ Power Gain

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2) \cdot |1 - S_{22} \cdot \Gamma_L|^2}$$

$$P_{in} = P_{in}(\Gamma_S, \Gamma_{in}(\Gamma_L), S)$$

$$P_L = P_L(\Gamma_S, \Gamma_{in}(\Gamma_L), S)$$

- The **actual** power gain **introduced** by the amplifier is less important because a higher gain may be accompanied by a **decrease** in input power (power actually drained from the source)
- We prefer to characterize the amplifier effect looking to the **power actually delivered to the load** in relation to the power **available from the source** (which is a constant)

# Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\ L}}{P_{av\ S}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2)}{|1 - S_{22} \cdot \Gamma_L|^2 \cdot (1 - |\Gamma_{out}|^2)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\ S}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2) \cdot (1 - |\Gamma_L|^2)}{|1 - \Gamma_S \cdot \Gamma_{in}|^2 \cdot |1 - S_{22} \cdot \Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

- **Unilateral transducer** power gain

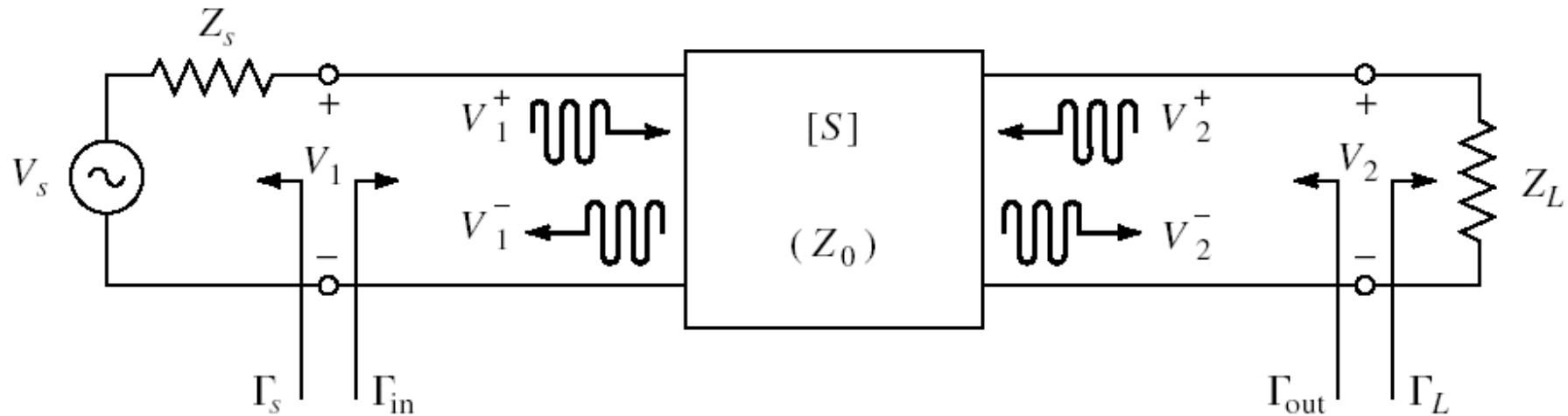
$$G_{TU} = |S_{21}|^2 \cdot \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

$$S_{12} \cong 0 \quad \Gamma_{in} = S_{11}$$



Input and output can be treated independently

# Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

# Contact

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